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RELIABILITY OF THERMAL STRESS IN TAPERING CIRCULAR BARS

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***Abstract*— In this particular article, we have to determine the reliability of thermal stresses in tapering circular bars whose body may be enlarge or indenture due to some raise or fall in the temperature of the body according to the addition. The thermal stresses or thermal strains may also be found out to first by finding out the amount of deformation due to change in temperature, and then by finding out thermal strain due to the deformation. The thermal stress may be found out the thermal strain. For such thermal stresses in tapering circular bars we are particularly interested in investigating the reliability by using the linear hazard model. It is also compared the results between the reliability of thermal stresses in bars if the ends of the bars are fixed to rigid supports and if the supports yield by an amount equal to ∆.**

***Keywords*—** Thermal stresses, temperature, reliability, coefficient of linear expansion (α), time, linear hazard model.

I. INTRODUCTION

Whenever there is some rise or fall in the temperature of a body, the body will do some expansion or contraction due to the variation of temperature. A little consideration will show that, if the body expands or contract freely due to the rise or fall of the temperature, no stresses are induced on the body. But if the deformation of the body is constrained (i.e. restriction), then stresses are developed on the body reckoned as thermal stress or temperature stress. The corresponding strain is reckoned as thermal strain or temperature strain.

Reliability is used for developing the equipment manufacturing and delivery to the user. A reliable system is one which operates according to our expectations. Reliability of a system is the probability that a system perform its intended purpose for a given period of time under stated environment conditions. In some cases system failures occur due to certain type of stresses acting on them. These types of system are reckoned as stress dependent models of reliability. These models now a days studied in many branches of science such as Engineering (Manufacturing, Production), Medicine, Pharmaceutical industries, R & D, and other various sectors, etc.

In assessing system reliability it is first necessary to define and categorize different modes of system failures. It is difficult to define failure in unambiguous forms. However a system's performance can deteriorate gradually over time and sometimes there is only a fine line between systems success and system failure. Once the system function and failure modes are explicitly stated reliability can be precisely quantified by probability statements.

II. METHODOLOGY (TACTICS)

The probability of failure as a function of time can be defined
$F(t) = (T \leq t)$, where $t \geq 0$ ....(2.1);

and T is a random variable denoting the failure time.
The Reliability function can be defined as the probability of success for the intended time is
$R(t) = 1 - F(t) = P(T > t)$ ....(2.2);
The Hazard function $h(t)$ can be defined as the limit of the failure rate as the interval approaches to zero. Thus the hazard function is the instantaneous failure rates is defined as
z(t) = f(t) / R(t) = f(t)/{1- F(t)} where f(t) dt = dF(t)………….(2.3);
h(t) = z(t) × σ……………………………………………………………… …(2.4);
when one kind of stress induced on the body.

III. THERMAL STRESS IN CIRCULAR TAPERING SECTIONS:-

a. Calculate the amount of deformation due to change of temperature with the assumptions that the bar is fixed at both ends.
b. Calculate the force (load) required to bring the deformed bar to the original length.
c. Calculate the stress and strain in the bar caused by the force.

Let
l = length of the bar;
d_1 = diameter of the larger end of the bar;
d_2 = diameter of the smaller end of the bar;
θ = Increase of temperature;
α = Co-efficient of linear expansion.

We know that as a result of the increase in temperature, the bar AB will tend to expand. But since it is fixed at both ends, therefore it will causes some compressive stress. We also know that increase in length due to rise of temperature. Therefore,

$\Delta l = l \times \alpha \times \theta$...(3.1);

Now let, P = load (or force) required to bring the deformed bar to the original length.

We know that decrease in the length of the circular bar due to load P is

$\Delta l = (4\times P\times l)/(\pi\times E\times d_1\times d_2)$……………………………………....(3.2);

Solving equation (3.1) and (3.2), we get-

$l \times \alpha \times \theta = (4\times P\times l)/(\pi\times E\times d_1\times d_2)$……………………….………(3.3);

Now,

$P = (\pi\times E\times d_1\times d_2\times\alpha\times\theta)/4$..(3.4);

Maximum stress

$(\sigma_{max}) = P/A = P/(\pi/4 \times d_2^2)$

$= (\pi\times E\times d_1\times d_2\times\alpha\times\theta)/4 \times (\pi/4 \times d_2^2)$

$= (\alpha \times \theta \times E \times d_1)/d_2$………………………………………..……(3.5);

.

Table:-I- Value of Coefficient of linear expansion for the different materials body

Serial Number	Material Body	Coefficient of linear expansion(α)
1	Aluminium(Al)	23×10^{-6} to 24×10^{-6}
2	Steel	11.5×10^{-6} to 13×10^{-6}
3	Cast iron, Wrought iron	11×10^{-6} to 12×10^{-6}
4	Copper, Brass, Bronze	17×10^{-6} to 18×10^{-6}

Table:-II-Value of Young's Modulus for the different materials body

Serial Number	Material Body	Modulus of Elasticity (E) in GPa (KN/mm^2)
1	Aluminium	60×10^3 to 80×10^3
2	Steel	200×10^3 to 220×10^3
3	Cast Iron	100×10^3 to 160×10^3
4	Wrought Iron	190×10^3 to 200×10^3
5	Copper	90×10^3 to 110×10^3
6	Brass	80 10^3 to 90×10^3

IV. LINEARLY INCREASING HAZARD MODEL:

When there is wear or deterioration of parts or components, the failure rate increases with time. The simplest model that we can consider in this category is one in which the Failure Rate increases linearly with time.

Let

$z(t)=kt$...(4.1);

where k is a constant .

Therefore, $h(t) = z(t) \times \sigma = kt \times \sigma$......................................(4.2);

When σ_{max} =MAXIMUM STRESS IN THE TAPERING CIRCULAR BAR IF THE ENDS OF THE BAR ARE FIXED TO RIGID SUPPORTS:

Stress in the bar $(\sigma_{max}) = (\alpha \times \theta \times E \times d_1)/d_2$ (from, equation 3,5)

Therefore, Reliablity of the bar R(t)

$= \exp[-(K\times\alpha\times\times\theta\times d_1\times t^2)/(2d_2)$ ………………………………………..(4.3);

Illustration-1:- A circular bar rigidly fixed at its ends which is 1.2 m long. It uniformly tapers from 100 mm at one end to 75 mm at the other end. What is the maximum stress induced in the bar, when its temperature raised through 25 K. Take E as 200GPa and $\alpha = 12 \times 10^{-6}$/K. Also, find the reliability of the bar for linear hazard model.

Solution-1:

Diameter at larger end (d_1) = 100 mm;

Diameter at smaller end (D_2) =75 mm;

Rise in temperature (θ) = 25K;

$E = 200 \times 10^3$ N/mm^2 ;

$\alpha = 12 \times 10^{-6}$/k.

We know that the maximum stress developed in the bar,

$\sigma_{max} = (\alpha \times \theta \times E \times d_1)/d_2$

Putting the values of the above equation, we get maximum stress = 80 N/mm^2 = 80 MPa

Reliability of the Bar R(t)

$= \exp[-(K\times\alpha\times\times\theta\times d_1\times t^2)/(2d_2)$

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$= \exp[-(0.01) \times 80 \times 0.1^2/2]$
$= 0.99996$

WHEN σ_{max} =MAXIMUM STRESS IN THE TAPERING CIRCULAR BAR IF THE SUPPORTS YIELD BY AN AMOUNT EQUAL TO Δ:

If the supports yield by an amount equal to δ, then the actual expansion that has taken place,

$\Delta l = (l \times \alpha \times t) - \delta$.

Now let, P = load (or force) required to bring the deformed bar to the original length.

We know that decrease in the length of the circular bar due to load P is –

$\Delta l = (\pi \times E \times d_1 \times d_2 \times \alpha \times \theta)/4$ ....(4.4);

Solving equation (2.5) and (2.6), we get-

$(l \times \alpha \times \theta) - \delta = (4 \times P \times l)/(\pi \times E \times d_1 \times d_2)$……………………......(4.5);

From 4.5, we get the formula of P, i.e.

$P = \pi E d_1 d_2 (l \times \alpha \times \theta - \Delta)/4$……………………………………(4.6);

Now, Maximum stress

$(\sigma_{max}) = (l \times \alpha \times \theta - \Delta) E d_1/d_2$ ……………………………….(4.7);

Therefore, Reliability of the Bar R(t)

$= \exp[-K (l \times \alpha \times \theta - \Delta) E \times d_1 \times t^2/2d_2$……………………….(4.8);

Illustration-2:- A circular bar rigidly fixed at its ends which is 1.2 m long. It uniformly tapers from 100 mm at one end to 75 mm at the other end. What is the maximum stress induced in the bar, when its temperature raised through 25 K, if the ends yield by 0.3 mm. Take E as 200GPa and $\alpha = 12 \times 10^{-6}$/K. Also, find the reliability of the bar for linear hazard model.

Solution-2:

Diameter at larger end (d_1) = 100 mm;
Diameter at smaller end (D_2) =75 mm;
Rise in temperature (θ) = 25K;
$E = 200 \times 10^3$ N/mm^2 ;
$\alpha = 12 \times 10^{-6}$/k.

We know that the maximum stress developed in the bar,

$\sigma_{max} = (l \times \alpha \times \theta - \Delta) E d_1/d_2$
$= 16$KN/mm^2

Reliability of the Bar R(t)
$= \exp[-K (l \times \alpha \times \theta - \Delta) E \times d_1 \times t^2/2d_2$
$= \exp[-(0.01) \times 16000]$
$= \exp[-0.008]$
$= 0.99203$

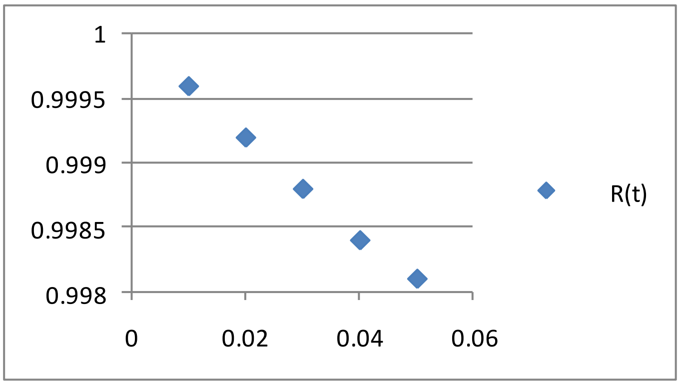
81 Graphical Representation of Reliability with the computation of Steel Tapering Circular Bar:

Reliability Computation of Tapering Circular Bar when σ=Stress in the rod if the ends do not yield when t = 0.01 and $\alpha = 12 \times 10^{-6}$/k :

Table-III

Constant(k)	Temperature θ(°K)	Modulus of Elasticity(E)	Stress(σ)	R(t)
0.01	25	200GPa	80MPa	0.99960
0.02	25	200GPa	80MPa	0.99920
0.03	25	200GPa	80MPa	0.99880
0.04	25	200GPa	80MPa	0.99840
0.05	25	200GPa	80MPa	0.99810
0.06	25	200GPa	80MPa	0.99760
0.07	25	200GPa	80MPa	0.99720
0.08	25	200GPa	80MPa	0.99680
0.09	25	200GPa	80MPa	0.99640
0.10	25	200GPa	80MPa	0.99600

Graph- I



Reliability Computation of Tapering Circular Bar when σ=Stress in the rod if the ends do not yield when k = 0.01 and $\alpha = 12 \times 10^{-6}$/k:

Table-IV

Time(t)	Temperature θ(°K)	Modulus of Elasticity(E)	Stress(σ)	R(t)
0.01	25	200GPa	80MPa	0.99960
0.02	25	200GPa	80MPa	0.99840

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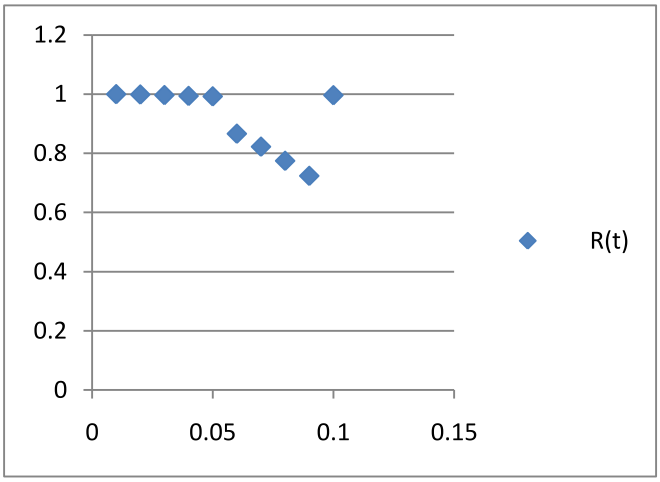
0.03	25	200GPa	80MPa	0.99640
0.04	25	200GPa	80MPa	0.99362
0.05	25	200GPa	80MPa	0.99219
0.06	25	200GPa	80MPa	0.86588
0.07	25	200GPa	80MPa	0.82201
0.08	25	200GPa	80MPa	0.77414
0.09	25	200GPa	80MPa	0.72325
0.10	25	200GPa	80MPa	0.99600

Graph-II

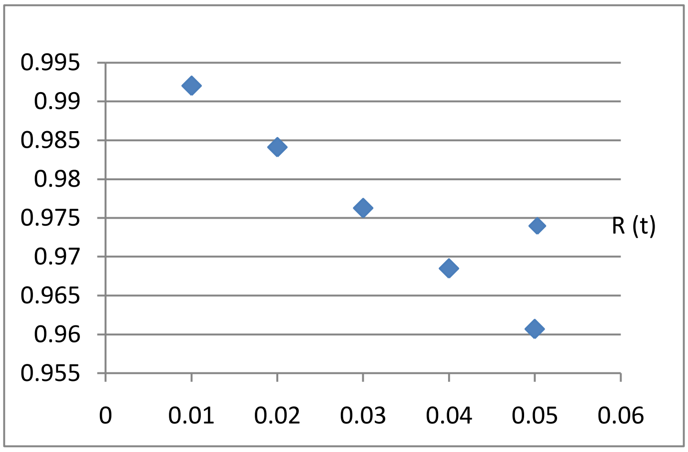


Reliability Computation of Tapering Circular Bar when σ=Stress in the rod if the ends yields by an amount Δ= 0.3 mm when t = 0.01 and $\alpha = 12 \times 10^{-6}$/k :

Table-V

Constant(k)	Temperature θ(°K)	Modulus of Elasticity(E)	Stress(σ)	R(t)
0.01	25	200GPa	16GPa	0.99203
0.02	25	200GPa	16GPa	0.98412
0.03	25	200GPa	16GPa	0.97628
0.04	25	200GPa	16GPa	0.96850
0.05	25	200GPa	16GPa	0.96078
0.06	25	200GPa	16GPa	0.95313
0.07	25	200GPa	16GPa	0.94553
0.08	25	200GPa	16GPa	0.93810
0.09	25	200GPa	16GPa	0.93053
0.10	25	200GPa	16GPa	0.92311

Graph-III

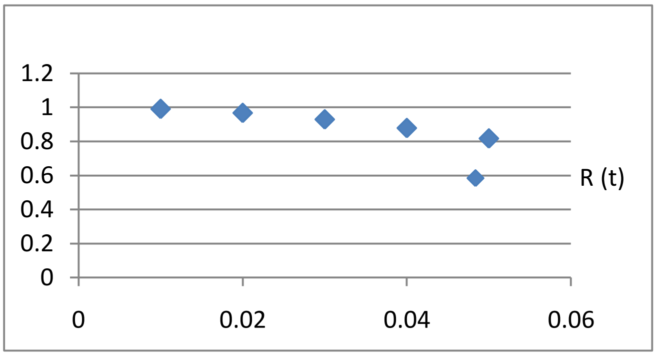


Reliability Computation of Tapering Circular Bar when σ=Stress in the rod if the ends yields by an amount Δ= 0.3 mm when K = 0.01 and $\alpha = 12 \times 10^{-6}$/k :

Table-VI

Time(t)	Temperature θ(°K)	Modulus of Elasticity(E)	Stress(σ)	R(t)
0.01	25	200GPa	16GPa	0.99203
0.02	25	200GPa	16GPa	0.96850
0.03	25	200GPa	16GPa	0.93053
0.04	25	200GPa	16GPa	0.87985
0.05	25	200GPa	16GPa	0.81873
0.06	25	200GPa	16GPa	0.74976
0.07	25	200GPa	16GPa	0.67570
0.08	25	200GPa	16GPa	0.59929
0.09	25	200GPa	16GPa	0.52309
0.10	25	200GPa	16GPa	0.44932

Graph- IV



V. CONCLUSION:-

Reliability of the thermal stresses or the thermal strains are set up initially by finding out the amount of deformation due to the change in temperature. Reliability calculations are obtained for the tapering circular bars when stress (σ) in the Rod (a) if the ends do not yield and (b) if the ends yield by an equal to amount Δ for various materials. Now as per the observation, when temperature remains constant, reliability decreases when constant term (k) increases. Similarly, temperature remains constant, but reliability decreases when time increases linearly. It is observed that the reliability is always depended on the coefficient of linear expansion. Therefore, the reliability varies with the certain parameters as well as the coefficient of linear expansion.

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