International Journal of Engineering Applied Sciences and Technology, 2018
Vol. 3, Issue 4, ISSN No. 2455-2143, Pages 14-20
Published Online August 2018 in IJEAST (http://www.ijeast.com)

AN ALGORITHM FOR INDEX NUMBERS USING AVERAGE WEIGHTED BASE.

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Abstract— **In this paper, we have used a new algorithm to find index number using a generalized base. The base used in the current work taken as the arithmetic mean of given set of quantities/prices for estimating price/quantity index numbers. We have tested the adequacy of new algorithm by several statistical tests and our algorithm successfully passed three of four tests. Also, the graphical representation of the estimations by the new algorithm have displayed for some particular examples.**

Keywords— **Statistic, Index number, Price, Quantity, Base, Average, Weight, Living cost, Adequacy, Graphical representation.**

I. INTRODUCTION

A. History of Index Number

The currently accepted view "Of the Value and Proportion of Metals Monetized with Genders in Italy before the Discovery of the Indies with the comparison of the Value and Proportion of the Times" in Custodians, Italian Writers of Economic Pollution, Modern Part, XIII. (pp.297-366), honor of inventing the device now commonly used to measure changes in the level of prices probably belongs to an Italian, G. R. Carli. In an investigation into the effect of the discovery of America upon the purchasing power of money, he reduced the prices paid for grain, wine, and oil in 1750 to percentages of change from their prices in 1500, added the percentages together, and divided the sum by three, thus making an exceedingly simple index number. Since his book was first published in 1764 by Freund et al. (1969), index numbers are over 150 years old. This view probably originated with Wesley Mitchell, who named Carli as the inventor of the index number. In point of fact by Kenney et al. (1962), this honor might be placed nearly a century earlier than Carli’s effort, and certainly 25 years before it, depending on how strictly one chooses to define the term index number.

If we give the term the broad meaning of a method or device which allows one to measure changes in aggregate price levels, we must consider a small book published by Rice Vaughan (1675). The Englishman Vaughan (1975) was concerned with the rise in prices which has occurred in his native land over the preceding century. He wished to separate the influence on this price rise of the debasement from the influence of the heavy influx of gold and silver into Spain from the East and West Indies. For this task he required a measure of the general price rise which had occurred in that century.

While Vaughan can be considered a forerunner of price index research, his analysis did not actually involve calculating an index. In 1707 Englishman William Fleetwood created perhaps the first true price index. An Oxford student asked Fleetwood to help show how prices had changed. The student stood to lose his fellowship since a fifteenth-century stipulation barred students with annual incomes over five pounds from receiving a fellowship Fleetwood, who already had an interest in price change, had collected a large amount of price data going back hundreds of years. Fleetwood proposed an index consisting of averaged price relatives and used his methods to show that the value of five pounds had changed greatly over the course of 260 years. He argued on behalf of the Oxford students and published his findings anonymously in a volume entitled “Chronicon Preciosum”.

B. Definition

Statistic is a quantity (such as a statistical median, quartile deviation, etc.), which is calculated from observed data by Fisher (1967). An index number is a statistic which assigns a single number to several individual statistics in order to quantify trends by Mitchell (1921) and Mudgett (1951). The best-known index in the United States is the consumer price index, which gives a sort of "average" value for inflation based on price changes for a group of selected products. The Dow Jones and NASDAQ indexes for the New York and American Stock Exchanges, respectively, are also index numbers.

C. Types of Index Numbers

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Price index Numbers: Price index numbers measure the relative changes in prices of a commodity between two periods. Prices can be either retail or wholesale. Price index numbers are useful to comprehend and interpret varying economic and business conditions over time.

Quantity Index Numbers: These types of index numbers are considered to measure changes in the physical quantity of goods produced, consumed or sold of an item or a group of items.
Methods of constructing index numbers: There are two methods to construct index numbers: Price relative and aggregate methods by Srivastava et al. (1989).

Value index number: An index number formed from the ratio of aggregate values in the given period to the aggregate values in the base period by Marriott (1990). Basically this is not an index number as ordinarily understood but a value relative.

Cost living Index Number: This is a theoretical price index that measures relative cost of living over time or regions, which has been elaborately reported by the "BLS Information", Glossary. U.S. Bureau of Labor Statistics Division of Information Services. February 28, 2008 and retrieved May 05, 2009. It is an index that measures differences in the price of goods and services, and allows for substitutions with other items as prices vary.

Although price indices and quantity indices are very similar in theoretical construction, it is very important to understand the difference between them. There is also a fundamental difference when it comes to measuring them. Changes in the price of an item are far easier to measure than changes in quantity (or value).

The quantities (or values) of items sold in different outlets may vary enormously, depending on many factors including the size and location of the outlets. Prices, by comparison, will not vary so much from outlet to outlet. It is also much easier to find out the price of an item, than the quantity that may have been sold during a month. Knowing price movements in a few outlets will be a good guide to the average movement overall. There will be a tendency for the price of similar items to change in a similar way. On the other hand, with some exceptions, quantities may vary widely and may be difficult to define. Knowing quantities (or values) from a few outlets or producers may not be a very good guide to the total.

D. Base of Index Numbers

The base period generally is known to be the period with which other periods are compared and whose values provide the weights for a price/quantity index. However, the concept of the "base period" is not a precise one and may be used to mean rather different things. Three types of base periods may be distinguished:

(i) The price/quantity reference period, that is, the period whose prices/quantities appear in the denominators of the price/quantity relatives used to calculate the index.

(ii) The weight reference period, that is, the period, usually a year, whose values serve as weights for the index. However, when hybrid expenditure weights are used in which the quantities/prices of one period are valued at the prices/quantities of some other period, there is no unique weight reference period.

(iii) The index reference period, that is, the period for which the index is set equal to 100.

The three reference periods may coincide but frequently do not.

E. Types of constructions of Index Numbers

Simple Index Number: A simple index number is a number that measures a relative change in a single variable with respect to a base. These types of Index numbers are constructed from a single item only.

Composite Index Number: A composite index number is a number that measures an average relative changes in a group of relative variables with respect to a base. A composite index number is built from changes in a number of different items.

Weighted Index number: If all commodities are not of equal importance. We assign weight to each commodity relative to its importance and index number computed from these weights is called weighted index numbers.

Aggregative method: The aggregate price/quantity of all items in a given year is expressed as a percentage of same in the base year, giving the index number.

$$\text{Aggregative Index number} = \frac{\text{Aggregated price/quantity in the given year}}{\text{Aggregated price/quantity in the base year}} \times 100$$

Relative method: The price/quantity of each item in the current year is expressed as a percentage of price/quantity in base year. This is called price/quantity relative and expressed as following formula.

$$\text{Relative Index number} = \frac{\text{Price/quantity in the given year}}{\text{Price/quantity in the base year}} \times 100$$

Simple aggregative method: In this method, the total of the prices of commodities in a given (current) years is divided by the total of the prices of commodities in a base year and expressed as percentage.

$$P_{0n} = \frac{\sum P_n}{\sum P_0} \times 100$$

Weighted aggregative method: These index numbers are the simple aggregative type with the fundamental difference that weights are assigned to the various items included in the index.

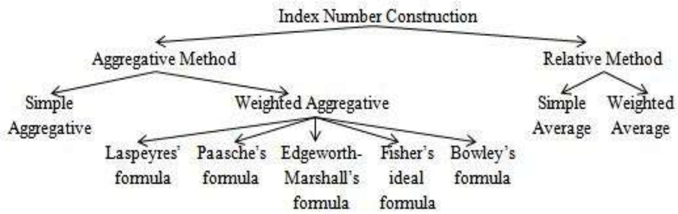
$$P_{0n} = \frac{\sum wP_n}{\sum wP_n} \times 100$$

Simple average relative method: In this method, we compute price/quantity relative or link relatives of the given commodities and then use one of the averages such as arithmetic mean, geometric mean, median etc. If we use arithmetic mean as average, then

$$P_{0n} = \frac{\sum\left(\frac{P_n}{P_0}\right)}{n} \times 100$$

Weighted average relative method: These are those index numbers in which price/quantity relative of each commodity and the sum of these products is divided by the sum of weights of all the commodities. In this method index can be calculated by taking arithmetic mean, geometric mean or even median as average.

$$P_{0n} = \frac{\sum w\left(\frac{P_n}{P_0}\right)}{\sum w} \times 100$$



F. Some weighted aggregative formula

If P_n & Q_n are the price & quantity [4] in the period n and P_0 & Q_0 are the price & quantity in the base period, then we get following weighted aggregative formulae.

Laspeyres' formula: The statistical index $\boldsymbol{P_L} = \frac{\sum P_n Q_0}{\sum P_0 Q_0} \times 100$

Paasches' formula: The statistical index $\boldsymbol{P_P} = \frac{\sum P_n Q_n}{\sum P_0 Q_n} \times 100$

Edgeworth-Marshall's formula: The statistical index

$$\boldsymbol{P_{E-M}} = \frac{\sum P_n\left(\frac{Q_0+Q_n}{2}\right)}{\sum P_0\left(\frac{Q_0+Q_n}{2}\right)} \times 100$$

Fisher's ideal formula: The statistical index

$$\boldsymbol{P_F} = \sqrt{P_L \times P_P} = \sqrt{\frac{\sum P_n Q_0}{\sum P_0 Q_0} \times \frac{\sum P_n Q_n}{\sum P_0 Q_n}} \times 100$$

Bowley's formula: The statistical index

$$\boldsymbol{P_B} = \frac{1}{2}(P_L + P_P) = \frac{1}{2}\left(\frac{\sum P_n Q_0}{\sum P_0 Q_0} + \frac{\sum P_n Q_n}{\sum P_0 Q_n}\right) \times 100$$

G. Characteristics of index numbers

1. Index numbers are specialized averages.
2. Index numbers measure the change in the level of a phenomenon.
3. Index numbers measure the effect of changes over a period of time.

H. Uses of Index number

Index numbers has practical significance in measuring changes in the cost of living, production trends, trade, and income variations. Index numbers are used to measure changes in the value of money. A study of the rise or fall in the value of money is essential for determining the direction of production and employment to facilitate future payments and to know changes in the real income of different groups of people at different places and times by Srivastava et al. (1989). Crowther designated, "By using the technical device of an index number, it is thus possible to measure changes in different aspects of the value of money, each particular aspect being relevant to a different purpose." Basically, index numbers are applied to frame appropriate policies. They reveal trends and tendencies and Index numbers are beneficial in deflating.

II. DERIVATION OF GENERAL WEIGHTED INDEX NUMBERS

Let P and Q represent the price and consumed quantity of certain commodity. Consider, P_i and Q_i are the values of prices and quantities of i^th commodity in different years. Then the average price and quantity of that commodity can be obtained as following

$$\bar{P} = \frac{\sum P_i}{N} \text{ and } \bar{Q} = \frac{\sum Q_i}{N}$$

Here N indicates the number of years those to be considered.

i. Weighted price index number with general weight

$$I_P^{(b,j)} = \frac{\sum P_j \bar{Q}}{\sum P_b \bar{Q}} \times 100$$

ii. Weighted quantity index number with general weight

$$I_Q^{(b,k)} = \frac{\sum \bar{P} Q_k}{\sum \bar{P} Q_b} \times 100$$

Here b indicates the base year and j & k indicate the corresponding years for those the price and quantity index numbers respectively are to be estimated.

III. TEST OF ADEQUACY

We have derived two following weighted index number with general weight. The details of test of adequacy for index numbers discussed by Satyadevi (2006).

Weighted price index number with general weight

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$$I_P^{(b,j)} = \frac{\sum P_j \bar{Q}}{\sum P_b \bar{Q}} \times 100$$

Weighted quantity index number with general weight

$$I_Q^{(b,k)} = \frac{\sum \bar{P} Q_k}{\sum \bar{P} Q_b} \times 100$$

A. Unit test

It requires that the method of constructing index should be independent of the units of the problem.

Here in both index numbers are the ratio of same unit with degree one. As a result the index numbers are unit less and are expressed in percentages. So, general weighted index numbers satisfy unit test of index numbers.

B. Time reversal test

According to Prof. Fisher the formula for calculating an index number should be such that it gives the same ratio between one point of time and the other, no matter which of the two times is taken as the base. In other words, when the data for any two years are treated by the same method, but with the base reversed, the two index numbers should be reciprocals of each other. If we interchange the base year and targeted year in above index numbers we have following.

Reversed weighted price index number with general weight

$$I_P^{(j,b)} = \frac{\sum P_b \bar{Q}}{\sum P_j \bar{Q}}$$

Reversed weighted quantity index number with general weight

$$I_Q^{(k,b)} = \frac{\sum \bar{P} Q_b}{\sum \bar{P} Q_k}$$

Now, $I_P^{(b,j)} \times I_P^{(j,b)} = \frac{\sum P_j \bar{Q}}{\sum P_b \bar{Q}} \times \frac{\sum P_b \bar{Q}}{\sum P_j \bar{Q}} = 1$

$$I_Q^{(b,k)} \times I_Q^{(k,b)} = \frac{\sum \bar{P} Q_k}{\sum \bar{P} Q_b} \times \frac{\sum \bar{P} Q_b}{\sum \bar{P} Q_k} = 1$$

Here we have omitted the factor 100 from each index numbers.

So, general weighted index numbers satisfy the time reversal test of index number.

C. Circular test

This test was suggested by Westerguard and C. M. Walsch. It is based on the shift ability of the base. Accordingly, the index should work in a circular fashion i.e., if an index number is computed for the period b on the base period a, another index is computed for period c on the base period b, and still another index number is computed for period a on the base period c. Then the product should be equal to one.

Let us consider $I_P^{(a,b)} = \frac{\sum P_b \bar{Q}}{\sum P_a \bar{Q}}$, $I_P^{(b,c)} = \frac{\sum P_c \bar{Q}}{\sum P_b \bar{Q}}$ and $I_P^{(c,a)} = \frac{\sum P_a \bar{Q}}{\sum P_c \bar{Q}}$.

Then we get $I_P^{(a,b)} \times I_P^{(b,c)} \times I_P^{(c,a)} = \frac{\sum P_b \bar{Q}}{\sum P_a \bar{Q}} \times \frac{\sum P_c \bar{Q}}{\sum P_b \bar{Q}} \times \frac{\sum P_a \bar{Q}}{\sum P_c \bar{Q}} = 1$

Similarly, we can consider $I_Q^{(d,e)} = \frac{\sum \bar{P} Q_e}{\sum \bar{P} Q_d}$, $I_Q^{(e,f)} = \frac{\sum \bar{P} Q_f}{\sum \bar{P} Q_e}$ and $I_Q^{(f,d)} = \frac{\sum \bar{P} Q_d}{\sum \bar{P} Q_f}$

Then we get $I_Q^{(d,e)} \times I_Q^{(e,f)} \times I_Q^{(f,d)} = \frac{\sum \bar{P} Q_e}{\sum \bar{P} Q_d} \times \frac{\sum \bar{P} Q_f}{\sum \bar{P} Q_e} \times \frac{\sum \bar{P} Q_d}{\sum \bar{P} Q_f} = 1$

Above process can be generalized for greater number of index numbers.

So, general weighted index numbers satisfy the circular test of index numbers.

D. Factor reversal test

It says that the product of a price index and the quantity index should be equal to value index. In the words of Fisher, just as each formula should permit the interchange of the two times without giving inconsistent results similarly it should permit interchanging the prices and quantities without giving inconsistent results which means two results multiplied together should give the true value ratio. The test says that the change in price multiplied by change in quantity should be equal to total change in value.

We have $I_P^{(b,r)} = \frac{\sum P_r \bar{Q}}{\sum P_b \bar{Q}}$ & $I_Q^{(b,r)} = \frac{\sum \bar{P} Q_r}{\sum \bar{P} Q_b}$

Also, we know the corresponding to above index numbers the value index number is $I_V^{(b,r)} = \frac{\sum P_r Q_r}{\sum P_b Q_b}$

Now we get

$$I_P^{(b,r)} \times I_Q^{(b,r)} = \frac{\sum P_r \bar{Q}}{\sum P_b \bar{Q}} \times \frac{\sum \bar{P} Q_r}{\sum \bar{P} Q_b} \neq \frac{\sum P_r Q_r}{\sum P_b Q_b} = I_V^{(b,r)}$$

$$\text{or, } I_P^{(b,r)} \times I_Q^{(b,r)} = I_V^{(b,r)}$$

So, general weighted index numbers not satisfy the factor reversal test of index numbers.

IV. APPLICATIONS AND GRAPHICAL REPRESENTATIONS

A. The prices (in dollars per pound) and quantities (annual per capita consumption in pounds) of three fruit items are given below.

Year→	1980		1985		1990	
Fruit↓	P	Q	P	Q	P	Q
Apples	0.692	19.2	0.684	19.3	0.719	19.6
Bananas	0.342	20.2	0.367	23.5	0.463	24.4
Oranges	0.365	14.3	0.533	13.4	0.571	12.4
Year→	1995		2000		2005	
Fruit↓	P	Q	P	Q	P	Q
Apples	0.835	18.9	0.927	17.5	0.966	16.1

Bananas	0.491	27.4	0.509	28.5	0.838	26.8
Oranges	0.625	12.1	0.638	11.7	0.491	10.6

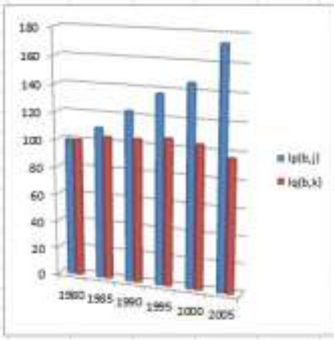
Let us consider 1980 as the base year. The average values of prices and consumed quantities of different fruits are obtained as below.

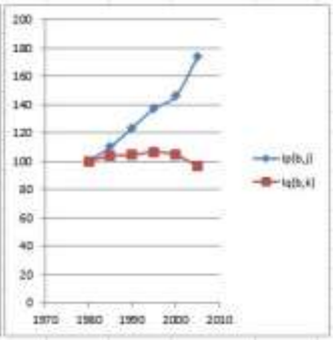
Average↓	Fruits→		
	Apples	Bananas	Oranges
P	0.804	0.502	0.537
Q	18.433	25.133	12.417

Now, the index numbers are as following.

Year	1980	1985	1990
$I_P^{(b,j)}$	100	109.9198	123.557
$I_Q^{(b,k)}$	100	103.7698	104.2389
Year	1995	2000	2005
$I_P^{(b,j)}$	137.1274	146.0503	173.7181
$I_Q^{(b,k)}$	106.5906	104.2206	96.4936

The graphical representations of above table are given below.





B. The prices for six food items (in dollars) and quantities consumed by a typical family are given below.

Year→		1995		2005	
Food↓		P	Q	P	Q
Bread	(Loaf)	0.77	46	1.98	56
Eggs	(Dozen)	1.85	27	2.98	20
Milk	(1 Liter)	0.88	99	1.98	130
Apples	(1 Kg)	1.46	26	1.75	40
Juice	(500 g)	1.58	40	1.73	42
Coffee	(250 g)	4.42	11	4.75	14
Year→		1995			
Food↓		P	Q		
Bread	(Loaf)	2.52	61		
Eggs	(Dozen)	3.82	18		
Milk	(1 Liter)	2.97	165		
Apples	(1 Kg)	2.02	55		
Juice	(500 g)	2.05	46		
Coffee	(250 g)	4.82	18		

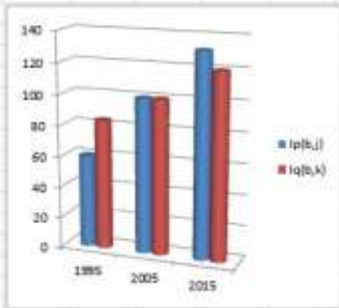
Let us consider 2005 as the base year. The average values of prices and consumed quantities of different fruits are obtained as below.

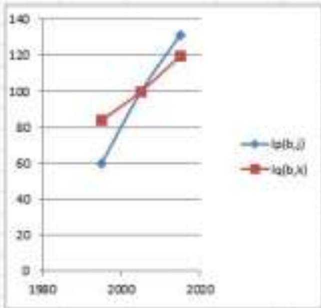
Food↓		Average→	
		P	Q
Bread	(Loaf)	1.757	54.3337
Eggs	(Dozen)	2.8837	21.6677
Milk	(1 Liter)	1.9437	131.333
Apples	(1 Kg)	1.7437	40.333
Juice	(500 g)	1.7877	42.667
Coffee	(250 g)	4.6637	14.333

Now, the index numbers are as following.

Year	1995	2005	2015
$I_P^{(b,j)}$	60.0534	100	131.5063
$I_Q^{(b,k)}$	83.9096	100	119.8648

The graphical representations of above table are given below.





C. The prices (in dollars per pound) and quantities (annual per capita consumption in pounds) of three commodities are given below.

Year→	1970		1980		1990	
Food↓	P	Q	P	Q	P	Q
A	2.25	25	2.55	30	2.70	35
B	1.75	15	2.05	25	2.30	40
C	3.25	30	3.65	40	3.90	55
Year→	2000		2010			

Food↓	P	Q	P	Q
A	3.15	45	3.25	60
B	2.55	50	2.65	55
C	4.15	65	4.20	75

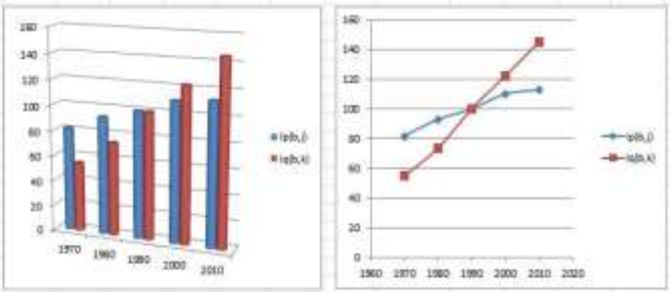
Let us consider 1990 as the base year. The average values of prices and consumed quantities of different fruits are obtained as below.

Food↓	Average→	
	P	Q
A	2.28	39
B	2.26	37
C	3.83	53

Now, the index numbers are as following.

Year	1970	1980	1990
$I_P^{(b,j)}$	81.7804	92.8607	100
$I_Q^{(b,k)}$	54.8011	73.5785	100
Year	2000	2010	
$I_P^{(b,j)}$	110.0856	112.6668	
$I_Q^{(b,k)}$	122.2669	145.1864	

The graphical representations of above table are given below.



(1). The prices for four food items (per meal in pence) and quantities consumed by peoples are given below.

Year→	2000		2003	
Meal↓	P	Q	P	Q
Fish & Chips	180	20	200	18
Pizza	160	22	180	17
Rice & Curry	220	18	240	20
Roast Lunch	240	15	260	13
Year→	2006		2009	
Meal↓	P	Q	P	Q
Fish & Chips	215	16	245	14
Pizza	205	13	235	10
Rice & Curry	270	23	310	27
Roast Lunch	280	12	300	10

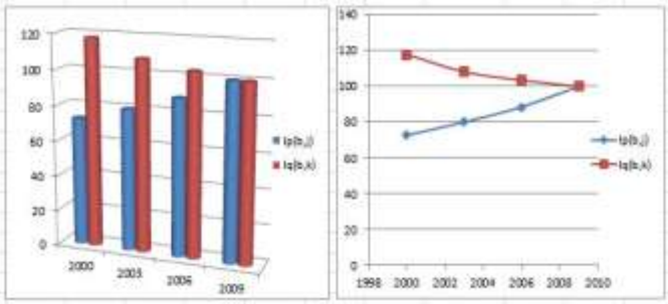
Let us consider 2009 as the base year. The average values of prices and consumed quantities of different fruits are obtained as below.

Meal↓	Average→	
	P	Q
Fish & Chips	210	17
Pizza	195	15.5
Rice & Curry	260	22
Roast Lunch	270	12.5

Now, the index numbers are as following.

Year	2000	2003	2006	2009
$I_P^{(b,j)}$	72.80642	80.09795	89	100
$I_Q^{(b,k)}$	117.8645	108.1793	103.4565	100

The graphical representations of above table are given below.



V.CONCLUSION

From the test of adequacy, we have seen that new algorithm of index numbers satisfies **unit test**, **time reversal test** and **circular test** but fails to satisfy **factor reversal test**. No previous index number algorithm rather than **Fishers ideal index number** satisfied more than two adequacy tests.

As the new algorithm based on arithmetic mean of data values of all given time intervals, it is more representative and logical. It is free of irregular movements of data values. The given examples have shown the applicability and accuracy of new algorithm. Also, the graphical representations have explained the eligibility of the current algorithm in the practical purposes.

Finally, in the new algorithm anyone can find index numbers for any specific time interval by using the common base obtained from arithmetic mean. So, it is time saving in case of multi-step index number estimations.

VI. ACKNOWLEDGEMENTS

This work is supported by the Institute of Natural Sciences, United International University, Dhaka and Department of Mathematics, Chittagong University, Chittagong.

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