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RESEARCH ON THE RELATIONSHIP BETWEEN ECONOMICAL PROFIT AND ENVIRONMENTAL POLLUTION OF IMPERFECT PRODUCTION INVENTORY CONTROL PROBLEM IN FUZZY-ROUGH ENVIRONMENT

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Abstract: It is well known fact that a country's economic growth leans on its natural resources. But the support of environmental capacity stands as back up in the operation of country's economic growth. In order to study the inherent relationship between the economic profit and environmental pollution, we are putting forward a new practice in this research area. Also numerous strategies have been set up to reduce the amount of polluting gases such as reliability development by modern technology on production system, rework on imperfective item which reduce the waste of defective item and reduce the economic loses. The said parameters are imprecise in nature for the lack of precise numerical information. In this paper, the imprecise parameters are taken fuzzy rough numbers. Then, a multi-objective production control inventory problem is formulated where the first objective function is for economical profit in maximizing form and second objective function is for calculating the amount of polluting gases in minimizing form in fuzzy-rough environment. The said imprecise problem is converted into equivalent crisp multi-objective problem by using fuzzy- rough mathematics. The said problem is converted into a single objective problem by weight sum method. Using optimal control theory, we have solved the optimal control problem and obtained the required optimal production and stock level in analytical forms. Then the numerical results of profit and amount of polluting gases are also obtained using MATHEMATICA software.

Keywords: Multi-Objective Problem, Fuzzy-rough variable, Imperfect Production with Rework, Reliability, Weight Sum Method

I. INTRODUCTION

While modern societies face growing concern about global environmental issues, developing countries are experiencing complex, serious and fast growing pollution problems of their own. The potent combination of industrialization, urban development and mass consumption trends is exacerbated by foreign companies operating with little regards for the impact on the local environment. To study this relationship between the economical profit and environmental pollution in uncertain environment we are trying to propose a new method in this research paper.

The relationship between economic profit and environmental pollution has received growing recognition throughout the world since 1970's. The said relationship has been extensively investigated and analyzed in some developed countries [cf. 1,2].The first and most important step in this endeavor is to analyze products impacts on environment with a holistic approach. This holism includes the analysis of the product life cycle from the very beginning up to the very end of it. Using this approach, ecological impacts of every little decision in various product stages such as product conceptualization, design, raw materials processing, manufacturing, assembly, warehousing, packaging, transportation, reusing and refurbishing is measured and considered in designing the product and the required operations. The industrial environmental pollution gases are produced when the production is in process and from disposal of defective item, etc which are called industrial solid waste (ISW) in production industry. Emission of carbon-di-oxide and the equivalent crbon-di-oxaide gases in the production period into the atmosphere and these gases are considered the principal components contributing to global warming. In this research paper, various policies have been implemented to reduce the pollution gases such as reliability development by modern technological on production system, re-work on imperfective item and extra objective function to calculate the amount of polluting gases. Modern technologies are executed by the industry to increase the reliability of the production system and decrease the carbon-di-oxide gasses emission in the environment. The production of imperfective items is increased with time and decreased with reliability parameters. The amount of waste of imperfective items is reduced by improving the system reliability and rework process. The defective items as a result of imperfect quality in production process were initially considered by Porteus [3] and later by several researchers such as Salameh and Jaber [4], Panda et al. [5], Sana [6], Pal et al. [7], Seluk and Agrali [8] and others. Recently, Andrade et al. [9] has managed the hazardous waste in the printing industry. Rework is a process using materials into new products to prevent waste of potentially useful materials, reduce the consumption of fresh raw materials, reduce energy usage, reduce air pollution in production inventory system. So, in this process, defective items are reworked. Lin et al. [10] developed a model on optimal replenishment policy for imperfect production inventory model with rework and backlogging. Next, Chiu et. al. [11] generalized the optimization of the finite production rate model with scrap, rework and stochastic machine breakdown. Recently, Hazari et al. [12], Maity [13] have developed an imperfect production inventory models with/ without rework. Different types of uncertainty such as randomness, fuzziness, roughness, bi-fuzzyness are common factors in any production inventory problem. In many cases, it is found that some inventory parameters involve fuzzy rough in nature. For example, the inventory related costs like holding cost, production cost, operating cost, pollution cost and selling price depend on several factors such as bank interest, stock amount, market situation, etc. which are uncertain in fuzzy rough sense. To be more specific, inventory holding cost is sometimes represented by a fuzzy rough number and it depends on the storage amount which may be imprecise and range within an interval due to several factors such as scarcity of storage space, market fluctuation, human estimation/ thought process. Das et al. [14] developed a two warehouse supply-chain model in imprecise environment. Islam and Roy [15] developed an economical production quantity model with flexibility and reliability consideration and demand dependent unit production cost under a space constraint in imprecise environment. Maity [16] developed a two warehouse production-inventory problem in imprecise environment. Xu and Zhao [17] have developed a multi-objective decision-making model with fuzzy rough co-efficient and its application to the inventory problem. Recently, Jana et al. [18] developed a bi-fuzzy production-recycling-disposal inventory problem with environment pollution cost via genetic algorithm. In this research paper, we are first time formulating a multi-objective production control inventory problem with environmental consciousness in uncertain environment. The first objective is for economical profit in maximizing form and second objective is for calculating the amount of polluting gases in minimizing form. The multi-objective problem is converted into a single objective problem by weight sum method and using optimal control theory [19], we solved the optimal control problem and obtain the required optimal production. Then using Mathematica 9.0 software, we solved the differential equation to obtain the optimal stock. The model is illustrated through numerical examples. Using the said software, sensitivity analysis for different parameters and results of objective functions are presented in tabular and graphical form.

II. THE FUZZY-ROUGH MATHEMATICS

Following Pawlak [20], Dubois and Prade [21], Demrc [22], Xu and Zhou [23] and Maity [13], some basic concepts, theorems and lemma on fuzzy rough sets are stated. The theorems and lamma are used to convert the fuzzy rough numbers to a equivalent crisp number.

Theorem-1: If fuzzy rough variables $\tilde{\bar{c}}_{ij}$ are defined as

$\tilde{\bar{c}}_{ij}(\lambda) = (\bar{c}_{ij1}, \bar{c}_{ij2}, \bar{c}_{ij3}, \bar{c}_{ij4})$ $with$ $\bar{c}_{ijt} \mapsto ([c_{ij2}, c_{ij3}, [c_{ij1}, c_{ij4}])$, for $i = 1,2, ¢¢¢; m; j = 1; 2; \ldots, n; t = 1,2,3,4, x = (x_1, x_2, \ldots, x_m), 0 \leq \tilde{\bar{c}}_{ij1} \leq \tilde{\bar{c}}_{ij2} < \tilde{\bar{c}}_{ij3} \leq \tilde{\bar{c}}_{ij4}$

. Then $E[\tilde{\bar{c}}_1^T x], E[\tilde{\bar{c}}_2^T x], \ldots, E[\tilde{\bar{c}}_n^T x]$ is equivalent to

$\frac{1}{16}\sum_{j=1}^{n}\sum_{t=1}^{4}\sum_{k=1}^{4} c_{1jtk}x_j, \frac{1}{16}\sum_{j=1}^{n}\sum_{t=1}^{4}\sum_{k=1}^{4} c_{2jtk}x_j, \ldots, \frac{1}{16}\sum_{j=1}^{n}\sum_{t=1}^{4}\sum_{k=1}^{4} c_{njtk}x_j$, (1)

Proof: The proof of the theorem is in reference Xu and Zhou [23] in page 308.

Theorem-2: If fuzzy rough variables $\tilde{\bar{a}}_{rj}$, $\tilde{\bar{b}}_r$ defined as follows,

$\tilde{\bar{a}}_{rj}(\lambda) = (\bar{a}_{rj1}, \bar{a}_{rj2}, \bar{a}_{rj3}, \bar{a}_{rj4})$ $with$ $\bar{a}_{rjt} \mapsto ([a_{rj2}, a_{rj3}], [a_{rj1}, a_{rj4}])$,

$\tilde{\bar{b}}_r(\lambda) = (\bar{b}_{r1}, \bar{b}_{r2}, \bar{b}_{r3}, \bar{b}_{r4})$ $with$ $\bar{b}_{rt} \mapsto ([b_{rt2}, b_{rt3}], [b_{rt1}, b_{rt4}])$,

for

$r = 1,2, \ldots, p, j = 1,2, \ldots, n, t = 1,2,3,4, 0 \leq a_{rt1} \leq a_{rt2} < a_{rt3} \leq a_{rt4}, 0 \leq b_{rt1} \leq b_{rt2} < b_{rt3} \leq b_{rt4}$.

then $E[\tilde{\bar{a}}_{rj}^T x] \leq E[\tilde{\bar{b}}_{rj}], r = 1,2, \ldots, p$ is equivalent to

$$\frac{1}{16}\sum_{j=1}^{n}\sum_{t=1}^{4}\sum_{k=1}^{4} a_{rjtk}x_j \leq \sum_{t=1}^{4}\sum_{k=1}^{4} b_{rtk}, \; r = 1,2, \ldots, p \quad (2)$$

Proof: The proof of the theorem is in reference Xu and Zhou [23] in page 316.

Lemma-1: Assume that ξ and η are the introduction of variables with finite expected values. Then for any real numbers a and b, we have

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta] \quad (3)$$

Proof: The proof of the Lemma is in reference Xu and Zhou [23] in page 308.

III. SINGLE-OBJECTIVE FU RO MODEL

Let us consider the following single-objective decision making model with fuzzy rough co-efficient:

$$\begin{cases} Max\,\{f(x,\xi)\} \\ s.t \begin{cases} g_r(x,\xi) \leq 0, r = 1,2, \ldots, p \\ x \epsilon X \end{cases} \end{cases} \quad (4)$$

where x is a n-dimensional decision vector, $\xi = (\xi_1, \xi_2, \xi_3, \ldots, \xi_n)$ is a Fu-Ro vector, $f(x,\xi)$ are objective function. Because of the existence of Fu-Ro vector ξ, problem (4) is not well-defined. That is, the meaning of maximizing $f(x,\xi)$ is not clear and constraints $g_r(x,\xi) \leq 0;\ r = 1,2, \ldots, p$ do not define a deterministic feasible set.

3.1 Equivalent crisp model for single objective problem with Fu Ro parameters:

For the single objective model (4) with Fu-Ro parameters, none can deal it directly, so first make it crisp equivalent. For this reason, we use theorems -1 & -2 and lemma -1 to transform the fuzzy rough model into Fu-Ro EVM i.e. crisp model and then solve it.

Based on the Theorems -1 & -2 and Lemma -1, the expected values of fuzzy rough events $f(x,\xi)$ and $g_r(x,\xi)$ is given by

$$\begin{cases} Max\{E[f(x,\xi)]\} \\ s.t \begin{cases} E[g_r(x,\xi)] \leq 0, r = 1,2, \ldots, p \\ x \epsilon X \end{cases} \end{cases} \quad (5)$$

where *x* is *n*-dimensional decision vector and ξ is *n*-dimensional fuzzy rough variable.

IV. OPTIMAL CONTROL FRAMEWORK

4.1. Assumption and Notation

To develop the reliability dependent imperfect production inventory control model, following assumptions and notations are used.

4.1.1. Assumptions

1. Production rate is function of time which is taken as control variable,
2. Demand rate is time dependent,
3. Imperfective rate is depended on reliability,
4. This is a single period inventory model with finite time horizon *T*,
5. Imperfect units occur only when the item is effectively produced and there is repair of imperfect units over the period [0*;* *T*],
6. Imperfect units are partially or fully reworked at a constant rate over the period [0*;* *T*]
7. Production cost is dependent on raw material cost and development cost which is fuzzy rough in nature,
8. Unit pollution cost due to the GHG emission and IWM policies in the production industry is constant,
9. Operating cost is also constant ,
10. The salvage value price of the finish stock is crisp constant,
11. Shortage is not allowed,
12. The inventory level is assumed to be continuous function of time,
13. The selling price and other inventory costs are fuzzy in nature.

4.1.2 Notations

T : time length of the cycle,

$U(t)$: production rate at time t which is a control variable,

$X(t)$: inventory level at time t,

$D(t)$: $(= D_0 + D_1 t)$ ynamic demand rate at time t where D_0 and D_1 are constants ,

β : rate of depreciation,

R : reliability parameter,

δ : defective parameter,

$\delta e^{(1-R)t}$: reliability dependent defective rate,

γ : constant rate of rework,

R_{min} : minimum value of the reliability parameter,

R_{max} : maximum value of the reliability parameter,

$\tilde{\tilde{M}}$: the costs like labor and energy which is fuzzy rough in nature,

$\tilde{\tilde{N}}$: the cost of technology, resource and design complexity for production when $R = R_{min}$ which is also fuzzy rough in nature,

$\tilde{\tilde{P}}_{u1}(R) = \tilde{\tilde{M}} + \tilde{\tilde{N}} e^{k \frac{R-R_{min}}{R_{max}-R}}$: fuzzy rough development cost dependent on reliability parameter,

$\tilde{\tilde{P}}_{u0}$: fuzzy rough raw material cost,

$\tilde{\tilde{P}}_{p}(R,t)$: fuzzy rough unit production cost which depend on production rate, raw material cost, development cost and wear tear cost,

$\tilde{\tilde{P}}_{op}$: fuzzy rough operating cost which is fuzzy rough in nature,

$\tilde{\tilde{P}}_{pl}$: fuzzy rough pollution cost which is fuzzy rough in nature,

$\tilde{\tilde{P}}_{rw}$: fuzzy rough rework cost per unit defective item ,

$\tilde{\tilde{P}}_{plg}$: fuzzy rough polluting gas which is fuzzy rough in nature,

$\tilde{\tilde{P}}_{rwg}$: fuzzy rough rework gas which is fuzzy rough in nature,

$\tilde{\tilde{P}}_{wg}$: fuzzy rough wastage gas which is fuzzy rough in nature,

$\tilde{\tilde{h}}$: fuzzy rough holding cost per unit item ,

$\tilde{\tilde{S}}$: fuzzy rough selling price per unit items sold ,

S_1 : salvage value price of finish stock at time *T*,

$p_i(t)$: adjoint function at time *t*, *i* = 1*;* 2*:*

4.2. Proposed Reliability dependent Imperfect Production-Inventory and pollution control Model in fuzzy rough environment

In this model, single-item reliability dependent imperfect production-inventory with rework is considered. Here, the items are produced at a variable rate *U*(*t*) of which $\delta e^{(1-R)t}$ (where $\delta e^{(1-R)t}$ <1) is reliability dependent imperfective rate. The reliability *R* decreases the imperfective rate. The defective item after rework is treated as disposal part. The demand rate *D*(*t*) of customers meet from the inventory.

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Therefore, the differential equation for stock level $X(t)$ representing above system during a fixed time-horizon, T is

$$\dot{X}(t) = \left(1-(1-\gamma)\delta e^{(1-R)t}\right)U(t) - D(t) \quad (6)$$

$$D(t) = D_0 + D_1 t \quad (7)$$

Where D_0 and D_1 are constants.

The unit production cost is considered as a function of produced-quantity. The raw material cost and development cost are fuzzy-rough in nature. So unit production cost is also fuzzy rough and is given by

where (.) denotes differentiation and the demand function $D(t)$ is dynamic demand rate during a fixed time-horizon, T is

$$\tilde{\tilde{P}}_p(R,t) = \tilde{\tilde{P}}_{u0} + \frac{\tilde{\tilde{P}}_{u1}(R)}{U(t)^2} \quad (8)$$

The final stock $X(T)$ is selling with salvage value price S_1 where $S_1 < \tilde{\tilde{S}}$ and the corresponding marginal revenue is $S_1 X(T)$.

Then the profit function is

$$\tilde{\tilde{J}}_1 = \int_0^T \left[\tilde{\tilde{S}}D(t) - \tilde{\tilde{h}}X(t) - \tilde{\tilde{P}}_{u0}U(t) - \frac{\tilde{\tilde{P}}_{u1}(R)}{U(t)} - \tilde{\tilde{P}}_{rw}\gamma\delta e^{(1-R)t}U(t) - \tilde{\tilde{P}}_{op}U(t) - \tilde{\tilde{P}}_{pl}U(t)\right]dt + S_1 X(T) \quad (9)$$

Whenever an item is produced certain amount of gas is emitted at the same time which acts as pollutant and creates pollution. For this reason we introduce another objective function for the created gas for an item is produced. We minimize the objective function. The items are produced at a variable rate $U(t)$ of which $\delta e^{(1-R)t}$ (where $\delta e^{(1-R)t} < 1$) is reliability dependent imperfective rate. Now $\tilde{\tilde{P}}_{plg}$ unit gas emitted per unit production which is in fuzzy rough in nature and $\tilde{\tilde{P}}_{plg}U(t)$ unit gas is emitted for total production. $\tilde{\tilde{P}}_{rwg}$ unit gas emitted for rework per unit rework which is also fuzzy rough in nature. But after the rework the whole thing cannot be turned out usable but resulting wastage. Now $\tilde{\tilde{P}}_{wg}$ unit gas emitted for wastage per unit wastage which is fuzzy rough in nature and $\tilde{\tilde{P}}_{wg}(1-\gamma)\delta e^{(1-R)t}U(t)$ is emitted gas for wastage. Then the pollution gas is emitted after the whole production is

$$\tilde{\tilde{J}}_2 = \int_0^T \left[\tilde{\tilde{P}}_{plg}U(t) + \tilde{\tilde{P}}_{rwg}\delta e^{(1-R)t}U(t) + \tilde{\tilde{P}}_{wg}(1-\gamma)\delta e^{(1-R)t}U(t)\right]dt \quad (10)$$

This is the realistic multi-objective inventory problem in which the profit function is maximized and the pollution gas function is minimized. i.e. The problem is

Maximize

$$\tilde{\tilde{J}}_1 = \int_0^T \left[\tilde{\tilde{S}}D(t) - \tilde{\tilde{h}}X(t) - \tilde{\tilde{P}}_{u0}U(t) - \frac{\tilde{\tilde{P}}_{u1}(R)}{U(t)} - \tilde{\tilde{P}}_{rw}\gamma\delta e^{(1-R)t}U(t) - \tilde{\tilde{P}}_{op}U(t) - \tilde{\tilde{P}}_{pl}U(t)\right]dt + S_1 X(T) \quad (11)$$

Minimize $\tilde{\tilde{J}}_2 = \int_0^T \left[\tilde{\tilde{P}}_{plg}U(t) + \tilde{\tilde{P}}_{rwg}\delta e^{(1-R)t}U(t) + \tilde{\tilde{P}}_{wg}(1-\gamma)\delta e^{(1-R)t}U(t)\right]dt$ (12)

Subject to (6) and (7)

4.3. The equivalent crisp production control problem

Following Maity [13], we convert the fuzzy rough imperfect production inventory and pollution control problem (6) - (7) and (11) - (12) into an equivalent crisp imperfect production and pollution control problem.

Maximize

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$$E(\tilde{\tilde{J}}_1) = \int_0^T \left[E(\tilde{\tilde{S}})D(t) - E(\tilde{\tilde{h}})X(t) - E(\tilde{\tilde{P}}_{u0})\,U(t) - \frac{E(\tilde{\tilde{P}}_{u1}(R))}{U(t)} - E(\tilde{\tilde{P}}_{rw})\gamma\delta e^{(1-R)t}U(t) - E(\tilde{\tilde{P}}_{op})U(t) - E(\tilde{\tilde{P}}_{pl})U(t) \right] dt + S_1X(T)$$

(13)

Minimize

$$E(\tilde{\tilde{J}}_2) = \int_0^T \left[E(\tilde{\tilde{P}}_{plg})U(t) + E(\tilde{\tilde{P}}_{rwg})\delta e^{(1-R)t}U(t) + E(\tilde{\tilde{P}}_{wg})(1-\gamma)\delta e^{(1-R)t}U(t) \right] dt$$

Subject to (6) and (7).

(14)

Following Maity and Maiti [24,25], we convert the above multi-objective problem in a single objective problem by weight sum method as

$$E(\tilde{\tilde{J}}) = w_1\,E(\tilde{\tilde{J}}_1) + w_2\,\left(-E(\tilde{\tilde{J}}_2)\right)$$

where $w_1 + w_2 = 1,\ w_1, w_2 \geq 0$.

4.4. Solution Method

The above control problem is solved using Pontryagin's maximum principle([19]). Using this principle, we construct the Hamiltonian function as

$$H = w_1\left(E(\tilde{\tilde{S}})D(t) - E(\tilde{\tilde{h}})X(t) - E(\tilde{\tilde{P}}_{u0})\,U(t) - \frac{E(\tilde{\tilde{P}}_{u1}(R))}{U(t)} - E(\tilde{\tilde{P}}_{rw})\gamma\delta e^{(1-R)t}U(t) - E(\tilde{\tilde{P}}_{op})U(t) - E(\tilde{\tilde{P}}_{pl})U(t) \right) + w_2\left(E(\tilde{\tilde{P}}_{plg})U(t) + E(\tilde{\tilde{P}}_{rwg})\delta e^{(1-R)t}U(t) + E(\tilde{\tilde{P}}_{wg})(1-\gamma)\delta e^{(1-R)t}U(t) \right) + p_1(t)\dot{X}(t)$$

$$= w_1\left(E(\tilde{\tilde{S}})D(t) - E(\tilde{\tilde{h}})X(t) - E(\tilde{\tilde{P}}_{u0})\,U(t) - \frac{E(\tilde{\tilde{P}}_{u1}(R))}{U(t)} - E(\tilde{\tilde{P}}_{rw})\gamma\delta e^{(1-R)t}U(t) - E(\tilde{\tilde{P}}_{op})U(t) - E(\tilde{\tilde{P}}_{pl})U(t) \right) + w_2\left(E(\tilde{\tilde{P}}_{plg})U(t) + E(\tilde{\tilde{P}}_{rwg})\delta e^{(1-R)t}U(t) + E(\tilde{\tilde{P}}_{wg})(1-\gamma)\delta e^{(1-R)t}U(t) \right) + p_1(t)\left(\left(1 - (1-\gamma)\,\delta e^{(1-R)t}\right) U(t) - D(t) \right)$$

(15)

where adjoint variable $p_1(t)$ is given by

$$\dot{p}_1(t) = -\frac{\partial H}{\partial X(t)}$$

(16)

Solving the above differential equations (15) - (16) with boundary condition $p_1(T) = S_1$, we get

$$p_1(t) = S_1 - w_1 E(\tilde{\tilde{h}})(T - t)$$

(17)

Putting the value of $p_1(t)$ in (15), we get, the Hamiltonian of the system

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$$H = w_1\left(E(\tilde{\tilde{S}})D(t) - E\left(\tilde{\tilde{h}}\right)X(t) - E(\tilde{\tilde{P}}_{u0})\,U(t) - \frac{E\left(\tilde{\tilde{P}}_{u1}(R)\right)}{U(t)} - E(\tilde{\tilde{P}}_{rw})\gamma\delta e^{(1-R)t}U(t) - E(\tilde{\tilde{P}}_{op})U(t) - E(\tilde{\tilde{P}}_{pl})U(t)\right) + w_2\left(E(\tilde{\tilde{P}}_{plg})U(t) + E(\tilde{\tilde{P}}_{rwg})\delta e^{(1-R)t}U(t) + E(\tilde{\tilde{P}}_{wg})(1-\gamma)\delta e^{(1-R)t}U(t)\right) + \left(S_1 - w_1E\left(\tilde{\tilde{h}}\right)(T-t)\right)\left(\left(1-(1-\gamma)\delta e^{(1-R)t}\right)U(t) - D(t)\right)$$

(18)

Therefore, our objective is to find out the optimal path of $U^*(t)$ such that H is maximum *i.e,* as $E(\tilde{\tilde{J}})$ is maximum.

Lemma 1 *The Hamiltonian function H has maximum in [0,T] for optimum production rate*

$$U^*(t) = \sqrt{\frac{A}{B}}\left[1 - \frac{C}{2B}t + \frac{3C^2+2BD}{12B^2}\,t^2 - \frac{3CD}{8B^2}\,t^3\right]$$

(19)

Where $A = w_1E\left(\tilde{\tilde{P}}_{u1}(R)\right)$

$$B = w_1\left(E(\tilde{\tilde{P}}_{u0}) + E(\tilde{\tilde{P}}_{op}) + E(\tilde{\tilde{P}}_{pl}) + \delta\gamma E(\tilde{\tilde{P}}_{rwg})\right) + w_2\left(E(\tilde{\tilde{P}}_{plg}) + \delta E(\tilde{\tilde{P}}_{rwg}) + \delta(1-\gamma)E(\tilde{\tilde{P}}_{wg})\right) + S_1\left(1-\delta(1-\gamma)\right) - w_1E\left(\tilde{\tilde{h}}\right)T\left(1-\delta(1-\gamma)\right)$$

$$C = w_1E(\tilde{\tilde{P}}_{rw})\delta(1-R) + w_2E(\tilde{\tilde{P}}_{rwg})\delta(1-R) + w_2E(\tilde{\tilde{P}}_{wg})\delta(1-\gamma)(1-R) + S_1\delta(1-\gamma)(1-R) + w_1E\left(\tilde{\tilde{h}}\right)T\delta(1-\gamma)(1-R) - w_1E\left(\tilde{\tilde{h}}\right)\left(1-\delta(1-\gamma)\right)$$

$$D = w_1E\left(\tilde{\tilde{h}}\right)\delta(1-\gamma)(1-R)$$

and we will deduce the optimal stock rate which is given by

$$X^*(T) = \sqrt{\frac{A}{B}}\left[1 - \frac{C}{4B}t^2 + \frac{3C^2+2BD}{12B^2}\,t^3 - \frac{3CD}{8B^2}t^4 - P\left(e^{(1-R)t} - 1\right) + Qt\,e^{(1-R)t} - S\,t^2e^{(1-R)t} + Y\,t^3e^{(1-R)t}\right] - \left(D_0t + D_1\frac{t^2}{2}\right)$$

(20)

where

$$P = \frac{\delta(1-\gamma)}{1-R} + \frac{\delta(1-\gamma)}{(1-R)^2}\frac{C}{B} + 2\frac{\delta(1-\gamma)}{(1-R)^3}\frac{3C^2+2BD}{4B^2} + 9\frac{\delta(1-\gamma)}{(1-R)^4}\frac{CD}{B^2}$$

$$Q = \frac{\delta(1-\gamma)}{1-R}\frac{C}{B} + \frac{\delta(1-\gamma)}{(1-R)^2}\frac{3C^2+2BD}{4B^2} + 9\frac{\delta(1-\gamma)}{(1-R)^3}\frac{CD}{B^2}$$

$$S = \frac{\delta(1-\gamma)}{1-R}\frac{3C^2+2BD}{4B^2} + 9\frac{\delta(1-\gamma)}{2(1-R)^2}\frac{CD}{B^2}$$

$$Y = 3\delta(1-\gamma)2(1-R)\frac{CD}{B^2}$$

Proof.

Differentiating Hamiltonian function H with respect to $U(t)$, we have

$$\frac{\partial H}{\partial U(t)} = \left(S_1 - w_1 E\left(\tilde{\tilde{h}}\right)(T - t)\right)\left(1 - (1-\gamma)\delta e^{(1-R)t}\right) - w_1\left(E\left(\tilde{\tilde{P}}_{u0}\right) - \frac{E\left(\tilde{\tilde{P}}_{u1}(R)\right)}{U(t)^2} + E\left(\tilde{\tilde{P}}_{op}\right) + E\left(\tilde{\tilde{P}}_{pl}\right) + E\left(\tilde{\tilde{P}}_{rw}\right)\gamma\delta e^{(1-R)t}\right) - w_2\left(E\left(\tilde{\tilde{P}}_{plg}\right) + E\left(\tilde{\tilde{P}}_{rwg}\right)\delta e^{(1-R)t} + E\left(\tilde{\tilde{P}}_{wg}\right)(1-\gamma)\delta e^{(1-R)t}\right)$$

(21)

For finding the optimal path $U^*(t)$,we know that

$$\frac{\partial H}{\partial U(t)} = 0 \qquad (22)$$

i.e.

$$\left(S_1 - w_1 E\left(\tilde{\tilde{h}}\right)(T - t)\right)\left(1 - (1-\gamma)\delta e^{(1-R)t}\right) - w_1\left(E\left(\tilde{\tilde{P}}_{u0}\right) - \frac{E\left(\tilde{\tilde{P}}_{u1}(R)\right)}{U(t)^2} + E\left(\tilde{\tilde{P}}_{op}\right) + E\left(\tilde{\tilde{P}}_{pl}\right) + E\left(\tilde{\tilde{P}}_{rw}\right)\gamma\delta e^{(1-R)t}\right) - w_2\left(E\left(\tilde{\tilde{P}}_{plg}\right) + E\left(\tilde{\tilde{P}}_{rwg}\right)\delta e^{(1-R)t} + E\left(\tilde{\tilde{P}}_{wg}\right)(1-\gamma)\delta e^{(1-R)t}\right) = 0$$

implies that,

$$U^*(t) = \sqrt{\frac{A}{B}}\left[1 - \frac{C}{2B}t + \frac{3C^2 + 2BD}{12B^2}\, t^2 - \frac{3CD}{8B^2}\, t^3\right]$$

, where *A*, *B*, *C*, *D* given above.

Using the above optimal production rate given in (19) in the equation (6), we obtain the optimal inventory level $X^*(t)$ in the interval [0,T] which is given by

$$X^*(T) = \sqrt{\frac{A}{B}}\left[1 - \frac{C}{4B}t^2 + \frac{3C^2+2BD}{12B^2}\, t^3 - \frac{3CD}{8B^2}t^4 - P\left(e^{(1-R)t} - 1\right) + Qt\, e^{(1-R)t} - S\, t^2 e^{(1-R)t} + Y\, t^3\, e^{(1-R)t}\right] - \left(D_0 t + D_1\frac{t^2}{2}\right)$$

,where *P*,*Q*,*S*,*Y* are given above.

V. NUMERICAL ILLUSTRATION

To illustrate the above reliability dependent production inventory model numerically, we consider the following input datas given in Table-5.1. The demand rate, optimum production rate and corresponding stock are shown in figure-1.

5.1. The fuzzy rough input datas

The fuzzy rough input datas for inventory parameters are given in Table -1.

5.2. Crisp input datas

The crisp input datas for inventory parameters are given in Table -2.

5.3. Optimum Result

For the above crisp input datas and using lemma-1, the optimum production rate $U^*(t)$, optimum stock level $X^*(t)$ and demand rate *D*(*t*) are shown in figure -1 respectively. Also optimum reliability, optimum profit and optimum gas emissions are $R^* = 0{:}663, J_1^* = 9464.92\$$ and $J_2^* = 143{:}441$ *litre* respectively. The development cost, profit function and gases emission with respect to reliability are shown in figures -2,-3 &-4 respectively.

Table 1. The fuzzy rough collected data

Fuzzy rough parameters	Fuzzy rough values ($, $, $, $)	with rough values ([$, $], [$, $])	Expectation of fuzzy rough parameters $
	($, $, $ $)	([$, $], [$, $])	$
$\tilde{\bar{h}}$	$(\bar{0.4}-2,\bar{0.4}-1,\bar{0.4}+1,\bar{0.4}+2)$	with $([0.38,0.42],[0.37,0.43])$	0.4
$\tilde{\bar{C}}_{u0}$	$(\bar{1.5}-2,\bar{1.5}-1,\bar{1.5}+1,\bar{1.5}+2)$	with $([1.48,1.52],[1.47,1.53])$	1.5
$\tilde{\bar{M}}$	$(\bar{430}-2,\bar{430}-1,\bar{430}+1,\bar{430}+2)$	with $([428,432],[426,434])$	430
$\tilde{\bar{N}}$	$(\bar{20}-2,\bar{20}-1,\bar{20}+1,\bar{20}+2)$	with $([8,12],[6,14])$	10
$\tilde{\bar{C}}_{op}$	$(\bar{0.5}-2,\bar{0.5}-1,\bar{0.5}+1,\bar{0.5}+2)$	with $([0.48,0.51],[0.47,0.53])$	0.5
$\tilde{\bar{C}}_{pl}$	$(\bar{0.7}-2,\bar{0.7}-1,\bar{0.7}+1,\bar{0.7}+2)$	with $([0.68,0.71],[0.67,0.73])$	0.7
$\tilde{\bar{C}}_{rw}$	$(1.0-2,1.0-1,1.0+1,1.0+2)$	with $([0.98,1.02],[0.97,1.03])$	1.0
$\tilde{\bar{C}}_{plg}$	$(\bar{0.45}-2,\bar{0.45}-1,\bar{0.45}+1,\bar{0.45}+2)$	with $([0.447,0.451],[0.43,0.47])$	0.45
$\tilde{\bar{C}}_{rwg}$	$(\bar{0.25}-2,\bar{0.25}-1,\bar{0.25}+1,\bar{0.25}+2)$	with $([0.24,0.26],[0.23,0.27])$	0.25
$\tilde{\bar{C}}_{wg}$	$(\bar{0.15}-2,\bar{0.15}-1,\bar{0.15}+1,\bar{0.15}+2)$	with $([0.14,0.15],[0.13,0.17])$	0.15
$\tilde{\bar{S}}$	$(\bar{75}-2,\bar{75}-1,\bar{75}+1,\bar{75}+2)$	with $([74,76],[73,77])$	75

Table 2. The crisp collected data

Parameters	w_1	w_2	S_1	k	R_{min}	R_{max}	δ	γ	T	D_0	D_1
Values	0.75	0.25	60$	0.6	0.45	0.9	0.03	0.65	12	1.5	1.3

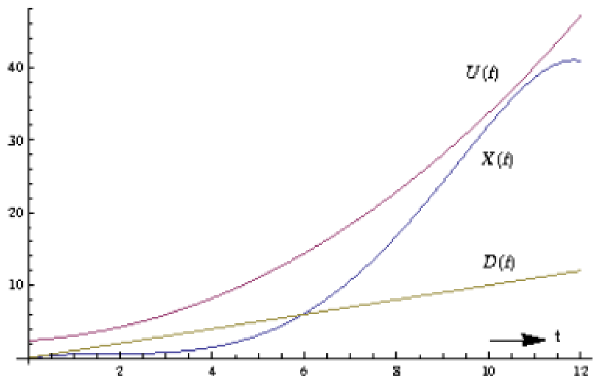

Fig. 1. Optimum production rate, optimum stock rate and demand rate versus time

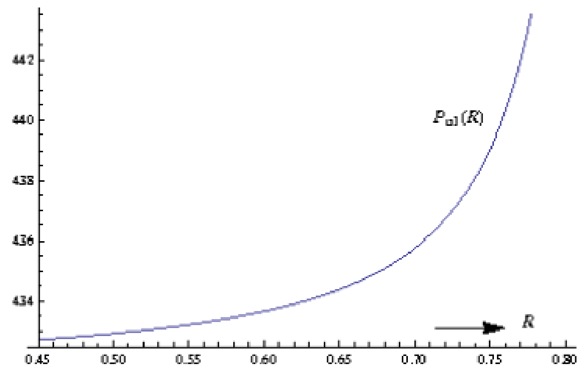

Fig. 2. Development cost versus reliability

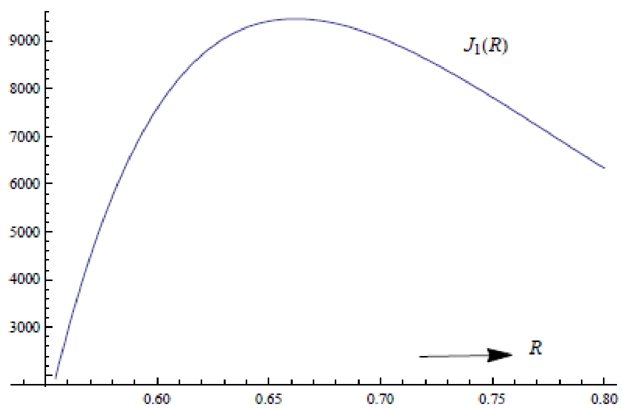

Fig. 3. Profit function versus reliability

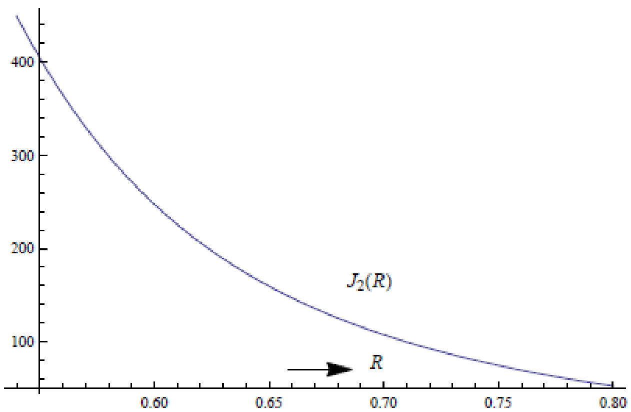

Fig. 4. Gases emission versus reliability

5.4. ***Sensitivity Analysis***

Table 3. Sensitivity analysis for γ

γ	0.6	0.65	0.7	0.75	0.8	0.85	0.9
$X^*(T)$	32.15	40.85	49.54	58.22	66.88	75.53	84.17
$J_1^*(\$)$	8957.02	9464.92	9972.12	10478.6	10984.4	11489.5	11993.8
$J_2^*(litre)$	144.71	143.44	142.17	140.91	139.64	138.38	137.12

The sensitivity analyses for rework γ are given in table-3. The table -4 is represented the sensitivity analysis of defective parameter. From the result in table -5, we observed that the development cost is increased for increased value of reliability and near the maximum value of reliability, development cost is very high shown in figure- 2, so the profit shown in figure-3 is maximum to the reliability value R = 0.663. From the results in Table -3, we observed that the rework has positive impact to the profit function as well as to the emission of the gas. When we will done maximum rework then the profit is increased and the emission of the gases are decreased.

Table 4. Sensitivity analysis for δ

δ	0	0.01	0.02	0.03	0.035	0.04
$X^*(T)$	101.73	81.48	61.19	40.85	30.67	20.47
$J_1^*(\$)$	13173.2	11939.9	10703.8	9464.92	8844.41	8223.19
$J_2^*(litre)$	95.99	111.78	127.59	143.44	151.38	159.33

From the results in Table -4, we observed that when the defective parameter δ is increased then the profit is decreased as well as the emissions of the gases are increased. The stock level is decreased for increased value of δ.

Table 5. Sensitivity analysis for reliability R

R	0.6	0.625	0.65	0.655	0.7	0.75
$c_1(\$)$	433.67	434	434.39	434.59	435.76	439.03
$X^*(T)$	14.24	33.81	40.77	41.04	32.47	10.43
$J_1^*(\$)$	7584.03	8890.23	9413	9448.4	9064.72	7818.87
$J_2^*(litre)$	247.36	197.41	159.53	153.08	107.49	74.56

From the results in Table-5, we observed that when the reliability is increased the profit is also increased and the emissions of the gases are decreased. But after a certain time suddenly the marginal revenue is decreased and for this reason the profit is also decreased.

VI. CONCLUSION

In this research paper, we have developed the correlation between economical profit and environmental pollution of imperfect production inventory control problem. To establish the relationship between the said economical profit and environmental pollution, various policies have been implemented to reduce the pollution gases such as reliability development by modern technology on production system, rework on imperfective item and extra objective function to calculate the amount of polluting gases. In this model we have observed that the reliability has positive effect to the profit function up to certain time and after this it has negative effect to the said profit function. At the same time, the amount of polluting gases (i.e the second objective function) are monotonically decreasing with respect to the reliability of the production system. So the decision maker of each industry should take to implement the reliability of the production system in a different way. On the basis of the analysis mentioned above, it has become a utmost necessity that the required environmental legislation should be placed at per with technologically developed countries to force the industry to invest in green technologies by the government. The govt. should also build and upgrade necessary infrastructure to enforce the environmental legislation effectively. And lastly the govt. should also ensure the uniform environmental legislation in all states/regions of the country to stop companies from shifting the dirty manufacturing to places with lax environmental legislation. The investigation can be extended for future research work on multi item optimal control problems where limits on *t* are imprecise/uncertain.

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