



NUMERICAL ANALYSIS OF BED LEVEL CHANGE OF JAMUNA RIVER USING PARABOLIC MODEL

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Abstract— The pride of Bangladesh is its rivers with one of the largest networks in the world. Bangladesh is located at the lower part of three mighty river basins and they are the Ganges, the Brahmaputra and the Meghna. The rivers have changed their courses frequently in the past. The old Brahmaputra course became a small tributary of the Brahmaputra-Jamuna River itself. Jamuna is one of the greatest rivers in the world ranking fifth in terms of discharge and eleventh in terms of drainage area. Jamuna is very dynamic in nature and the sufferings it causes to the people along with damages to national properties. Getting idea on river response due to dredging is a complex task. A research was undertaken to develop a parabolic model of the Jamuna river to understand the bed level changes due to dredging. This study can play an important role to cope with the challenges of bank erosion, sediment transport, flood control and navigation etc. The practical applications like flood and sediment routing, aggradation and degradation due to natural and human interference can be estimated to a certain extent by using numerical analysis of parabolic model. However, the use of the model for such a large river might have some restrictions. But it is worth mentioning that in this research both analytical and numerical analysis was done to overcome shortcomings.

Keywords— Jamuna River, dredging, bed level, numerical analysis, parabolic model.

I. INTRODUCTION

Bangladesh is a riverine country where most of the river is originated in the glacier of Jimayanzhong at the north foot of the Himalayas Mountain and sharply turns southward thus forming the famous yalutsangpojiang great bend. The total number of rivers of this country is almost 311 where 53 rivers come from India and 4 rivers from Myanmar [11]. Among them, Jamuna River is one of the major rivers in Bangladesh which has very crucial or effective sufferings to the people as well as national properties. Within Bangladesh, the length of the Jamuna River is approximately 240 km along the right bank and 220 km along the left bank. The catchment area of

Jamuna River is about 47,000 sq. km in Bangladesh. The average annual flood of the Jamuna river is almost 60,000 m³/s and the discharges vary from 4,000 m³/s to 12,000 m³/s in low flow season. The depth-averaged velocity is 3 m/s during flood and the annual sediment flow is about 65 crore tons which also indicate the bed level situation [6]. The Jamuna River catchment supplies large quantities of sediment from the actively uplifting mountains in the Himalayas. It carries a heavy sediment load, estimated to be over 650 million tons annually and most of the suspended sediment load is in the silt size class but around 15 to 25 percent is sand for bed load [4].

Nowadays for morphological analysis of a river, different types of analysis are invented such as Numerical Analysis, Sediment Transport Analysis, Helical Flow Analysis, Constant Discharge, Flow Resistance Models etc. Among these models, Numerical Analysis is using very widely all over the world to predict the morphological effect of engineering design consideration such as Hydraulic Structure, Barrages and Dams, Deeping of navigation and climate change [1]. The impact of river morphological changes has strong influence over social and economic conditions. Deformed char lands and river bank line shifting have effective socio-environmental issue [2]. In this study, numerical analysis has been done on change of bed level due to dredging of the river and analyze the river responses due to dredging on a selected reach of the Jamuna River by preparing a morphological model of this river [10]. When the river bed is lower over a period of time, then it's called Degradation. It has many causes like land use changes, river cutoffs, major flood, dam construction on the river or its tributaries, flow modification by reservoir regulation, dikes and revetment construction for navigation etc [7]. Depending upon the cause of degradation, river bed degradation can proceed downstream as well as upstream. The causes of downstream degradation are increase in water discharge, decrease in size of bed material, and decrease in bed material discharge. The upstream degradation can occur as a result of natural river behavior or by man-made changes [3]. Numerical modelling of river is very complex because of the strong interrelation among mechanisms that act at different spatial and temporal scales. Numerical models are today



extensively used to predict the morphological impact such as hydraulic structures, barrages and dams and climate change. In order to correctly predict the morphological impact of Jamuna river, it is necessary to reproduce the basic morphological features observed in nature [12]. Actually, the changes of bed level are significantly influence the turbulent characteristics of flow [8]. Bed level changes also influence the flood condition of a river. Channel width increases when bed degradation decreases. Continuous bed degradation affects channel width variation and bank stability [5].

II. REVIEW OF PARABOLIC MODEL

The Application of a parabolic model is valid if the effect of backwater is neglected [9]. This means that the water motion is to be considered as steady and uniform i.e., $\frac{\delta u}{\delta t} = 0$, $\frac{\delta u}{\delta x} = 0$, $\frac{\delta h}{\delta x} = 0$

Based on this assumption, it is possible to reduce $S^{-1} \frac{\delta u}{u \delta x} + \frac{\delta h}{\delta x} + \frac{\delta Z_b}{\delta x} = S_0 - \frac{u^3}{c^2 q}$ to a following linearized form,

$$\frac{\delta Z_b}{\delta x} = \frac{3S_0}{h_0} (\eta - Z_b) = A_0 (\eta - Z_b) \dots \dots \dots (1)$$

Differentiating of equation (1) with respect to x,

$$\frac{\delta^2 Z_b}{\delta x^2} = A_0 \left(\frac{\delta \eta}{\delta x} - \frac{\delta Z_b}{\delta x} \right) \dots \dots \dots (2)$$

$$\text{We know, } \frac{\delta Z_b}{\delta t} + C_0 \frac{\delta Z_b}{\delta x} - C_0 \left(\frac{\delta \eta}{\delta x} \right) = 0$$

Eliminating $\frac{\delta \eta}{\delta x}$ using this equation. So, equation (2) can be written as,

$$\frac{\delta^2 Z_b}{\delta x^2} = A_0 \left(\frac{\delta Z_b}{\delta t} \cdot \frac{1}{C_0} + \frac{\delta Z_b}{\delta x} - \frac{\delta Z_b}{\delta x} \right)$$

$$\rightarrow \frac{\delta^2 Z_b}{\delta x^2} = A_0 \left(\frac{\delta Z_b}{\delta t} \cdot \frac{1}{C_0} \right)$$

$$\rightarrow \frac{\delta Z_b}{\delta t} - \frac{C_0}{A_0} \cdot \frac{\delta^2 Z_b}{\delta x^2} = 0$$

$$\rightarrow \frac{\delta Z_b}{\delta t} - K_0 \cdot \frac{\delta^2 Z_b}{\delta x^2} = 0 \quad [\text{Here, } K_0 = \frac{C_0}{A_0}]$$

$$\therefore \frac{\delta Z_b}{\delta t} - K_0 \cdot \frac{\delta^2 Z_b}{\delta x^2} = 0 \quad [\text{Proved}]$$

This expresses the parabolic model for morphological computations.

III. DATA COLLECTION AND DATA ANALYSIS

- Upstream flow during flood, $Q = 50000 \text{ m}^3/\text{s}$
- Downstream water level, $h = 8 \text{ m}$
- Width of channel, $b = 10,000 \text{ m}$
- Length of channel, $L = 240,000 \text{ m}$
- Grain size, $D_{50} = 0.22 \text{ mm}$
- Initial slope, $S_0 = 0.00007$
- Chezy's coefficient, $C = 70 \text{ m}^{1/2} / \text{s}$
- $a = 0.02$
- $b = 2$
- Velocity of water, $u = 2.7 \text{ m/s}$
- Sediment transport rate, $q_s = a u^b = 0.02 \times (2.7)^2 = 0.14 \text{ m}^3 / \text{s/m}$
- Upstream flow rate during flood, $q = \frac{50000}{10000} = 5 \text{ m}^3 / \text{s/m}$
- $t = 25\text{s}, 50\text{s}, 100\text{s}, 500\text{s}, 1000\text{s}, 2000\text{s}, 5000\text{s}, 50000\text{s}$
- $\Delta z = 1 \text{ m}$

$$\Delta x = 10,000 \text{ m}$$

$$X = L = 240,000 \text{ m}$$

Analytical Solution Parabolic Model:

$$K_0 = \frac{C_0}{A_0}$$

$$C_0 = abq^b h^{-b-1} = \frac{0.02 \times 2 \times 5^2}{h^3} = \frac{1}{h^3} = \frac{1}{8^3} = 0.00195$$

$$A_0 = \frac{3S_0}{h} = \frac{3 \times 0.00007}{8} = 0.00002625$$

$$K_0 = \frac{0.00195}{0.00002625} = 74.3$$

$$\Delta t < \frac{\Delta x^2}{2 \cdot K_0}$$

$$< \frac{1000^2}{2 \times 74.3}$$

$$< 672947.5$$

$$\Delta t = 670000 \text{ sec} = 186.1 \text{ hour} = 186 \text{ hour}$$

Table -1 Analytical Calculation of Bed Level (EXCEL output)

Space	Analytical Bed level calculation for different time and space			
	t=0hr	t=5580hr	t=7440hr	t=11160hr
0	-1	-1.1E-05	-0.00014	-0.0019
10000	-1	-2.6E-05	-0.00027	-0.00291
20000	-1	-5.7E-05	-0.00049	-0.00441
30000	-1	-0.00012	-0.00087	-0.00657



40000	-1	-0.00025	-0.00152	-0.00964
50000	-1	-0.00051	-0.0026	-0.01393
60000	-1	-0.00099	-0.00433	-0.01983
70000	-1	-0.00186	-0.00705	-0.02779
80000	-1	-0.00341	-0.01121	-0.03838
90000	-1	-0.00604	-0.01742	-0.05222
100000	-1	-0.01039	-0.02648	-0.07
110000	-1	-0.01734	-0.03934	-0.09248
120000	-1	-0.02807	-0.05716	-0.12041
130000	-1	-0.04408	-0.08123	-0.15455
140000	-1	-0.06721	-0.11295	-0.19559
150000	-1	-0.0995	-0.1537	-0.2441
160000	-1	-0.14313	-0.20477	-0.30049
170000	-1	-0.20012	-0.26719	-0.36496
180000	-1	-0.27213	-0.34158	-0.43744
190000	-1	-0.36011	-0.42804	-0.51756
200000	-1	-0.4641	-0.52606	-0.60468
210000	-1	-0.58294	-0.63441	-0.69782
220000	-1	-0.71432	-0.75123	-0.79576
230000	-1	-0.85477	-0.87406	-0.89702
240000	-1	-1	-1	-1

7	-0.0098	-0.07809	-0.1235	-0.20467
7	-0.0105	-0.10747	-0.16162	-0.25126
7	-0.0112	-0.15643	-0.21664	-0.30972
7	-0.0119	-0.20887	-0.27538	-0.3718
7	-0.0126	-0.2867	-0.35346	-0.44585
7	-0.0133	-0.36866	-0.43547	-0.52331
7	-0.014	-0.47713	-0.53591	-0.61115
7	-0.0147	-0.58941	-0.6397	-0.70168
7	-0.0154	-0.72216	-0.75687	-0.79929
7.5	-0.0161	-0.85721	-0.87598	-0.89838
8	-1	-1	-1	-1

Table 2 shows the time required to lower the bed and it is formatted by using the following equation in excel. The table is developed for numerical solution of lowering the Jamuna river bed at the mouth of the river.

$$Z^{i^{n+1}} = Z^{i^n} + \frac{K_0 \cdot \Delta t}{\Delta x^2} (Z^{i^{n+1}} - 2Z^{i^n} + Z^{i^{n-1}})$$

IV. RESULT AND DISCUSSION

The simulated model results are presented in the following chart diagram.

$$Z(x, t) = -\Delta z * \operatorname{erfc} \left[\frac{x}{2\sqrt{Kt}} \right]$$

Here, erfc is the error function.

Numerical Solution of Parabolic Model:

Initial Condition is the initial bed level.

Downstream boundary condition is fixed as -1.

Upstream boundary condition is taken as equilibrium boundary

that is $\frac{\partial z}{\partial t} = 0$. Numerically it can be written as $\frac{Z^{i^{n+1}} - Z^{i^{n-1}}}{2\Delta x}$

$$Z^{i^{n+1}} = Z^{i^{n-1}}$$

$$Z^{i^{n+1}} = Z^{i^n} + \frac{K_0 \cdot \Delta t}{\Delta x^2} (Z^{i^{n+1}} - 2Z^{i^n} + Z^{i^{n-1}})$$

Table -2 Numerical Calculation of Bed Level (EXCEL output)

Water level(m) h(m)	Numerical Bed level calculation for different time and space			
	t=0hr	t=5580hr	t=7440hr	t=11160hr
7	0	-0.00304	-0.00371	-0.00781
7	-0.0007	-0.00313	-0.0038	-0.00801
7	-0.0014	-0.00327	-0.00411	-0.00933
7	-0.0021	-0.00357	-0.0046	-0.01111
7	-0.0028	-0.00399	-0.00556	-0.01449
7	-0.0035	-0.00457	-0.00676	-0.01853
7	-0.0042	-0.00553	-0.00908	-0.02516
7	-0.0049	-0.00667	-0.01179	-0.03281
7	-0.0056	-0.00896	-0.01697	-0.04456
7	-0.0063	-0.01158	-0.02287	-0.05784
7	-0.007	-0.01724	-0.03359	-0.07724
7	-0.0077	-0.02358	-0.04559	-0.09879
7	-0.0084	-0.03665	-0.06594	-0.12871
7	-0.0091	-0.05111	-0.08837	-0.16145

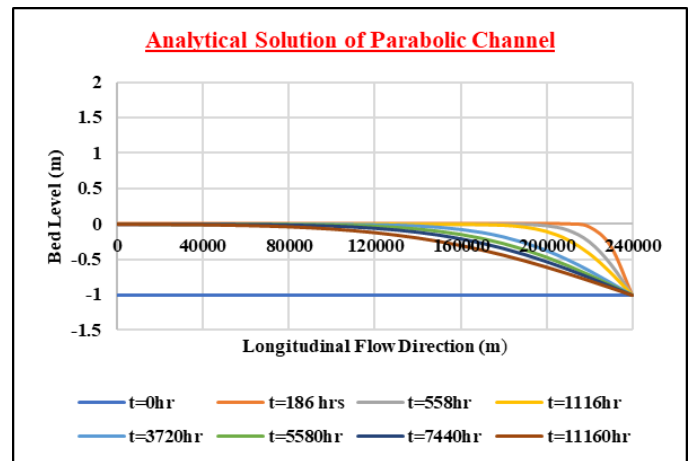


Figure 1: Bed level change of Jamuna River (Analytically)

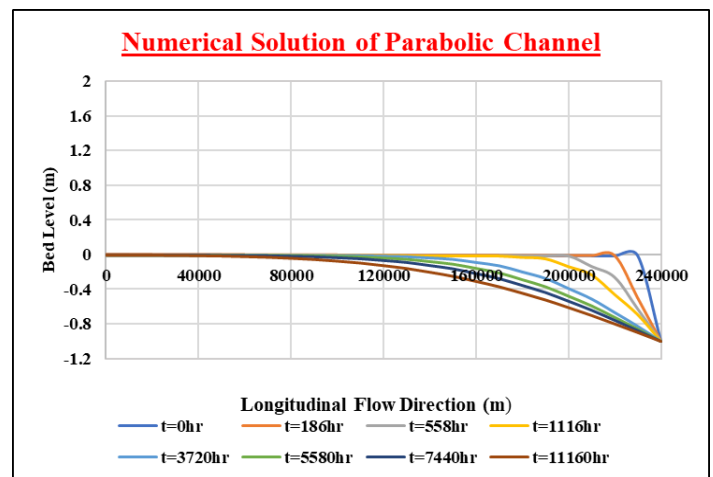


Figure 2: Bed level change of Jamuna River (Numerically)



The graphical representations show the change of bed level with time. At $\Delta t=0$, level drops by Δz . The slope is gradually decreasing when water depth reduces to make an equilibrium condition. With the help of the parabolic model, morphological time scale is determined. With the increase of time, the slope becomes flatter. For 1 m degradation of the river mouth, it takes 11160 hours which means more than 1 year. Analytically graphical representation is smoother as friction is neglected there. But numerical simulations of river morphology are extremely complicated. Because numerical solution covers time and length scales ranging from second to years and from mm to km. In numerical simulations the time and length scales are defined by the model resolution. So, there are some errors in the transition of the curve.

V. CONCLUSION

In this study the change of bed level bed was investigated analytically and numerically. The comparison shows the degradation of the bed level with morphological time scale. The analytical solution is obtained from the laplace transformation. The degradation takes place in the river mouth and for the numerical solution finite element method is used. The parabolic model is valid if the effect of backwater is neglected and the water motion is to be considered as steady and uniform. But practically it is quite difficult because a river has several parameters which influences the flow of that river. With the increasing of dredging depth, velocity increases along the dredged channel and decreases along the bank. The bed materials along the bank stay at the threshold point of erosion during average flood year of the Jamuna River while at higher dredging depth condition, the velocity along bank decreases in such an amount so that it becomes lower than the critical velocity and the bank becomes non erosive zone [10]. The practical applications, such as flood and sediment routing, aggradation and degradation due to natural and human interference, can be estimated to a certain extent by using numerical model. The schematization of the Jamuna river into a parabolic model is quite difficult as the length of the river is too large and that's why some errors occurs in the modeling.

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