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# STUDY OF MHD FLOW OVER A NON-LINEAR STRETCHING SHEET IN THE PRESENCE OF HEAT GENERATION/ABSORPTION

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**Abstract - The present paper deals with the study of momentum, heat and mass transfer characteristics in a MHD flow over a non-linear stretching sheet. The stretching of the sheet is assumed to be non-linearly proportional to the distance from slit. Two different temperature condition are studied (i) the sheet with prescribed surface temperature (PST) and (ii) the sheet with prescribe wall heat flux (PHF). The basic boundary layer equations for momentum and heat transfer, which are non-linear partial differential equation, are converted by means of similarity transformation. The resulting non-linear momentum equation is solved exactly. The energy equation in the presence of internal heat generation or absorption is a differential equation with variable coefficients which is transformed to a confluent hyper geometric equation. The solution and heat characteristics are obtained in terms of Kummer's function. The effect of various parameter, temperature profile and wall heat transfer are presented graphically.**

**Keywords:** Stretching sheet, Magnetic field, Kummer's function, confluent hypergeometric equation.

## I. INTRODUCTION

In recent years the studies of boundary layer flows on Newtonian fluids on stretching surfaces have great deal of applications in the fields of rolling and manufacturing of plastic film, artificial fibers, screwing, annealing and tinning of copper wires. Extrusion of a material and heat-treated materials that travel between feed and wind-up rollers or on conveyor belts these practical application and manufacturing process has a great attention due to flow of an incompressible fluid and heat transfer phenomenon over a stretching sheet. Water and air which are convectional fluids mostly used as cooling medium. For some sheet the rate of heat exchange due to above fluids are not suitable. Magnetic field is abundantly used in controlling flow kinematics for its easy use and intrusive quality. The rate of stretching electrically conducting fluid and use of magnetic fields can control the rate of cooling.

The classical problem of the stretching of the sheet as discussed by Sakiadis[1961], Crane[1970] and Magyari, E and many others involves the assumption of linear stretching. The impact of heat and mass transfer on free convective flow over a porous vertical plate in the presence of transverse magnetic field discussed by Mankinde and Ogulu[2008]. A numerical solution for the heat transfer boundary layer flow of non-Newtonian fluid past a stretching sheet was obtained by Mukhopadhyay. Rashidi et al.[2014] solved the heat and mass transfer effects in two dimensional MHD steady flow of visco elastic fluid.

The flow characteristics of fluid across a nonlinear stretching sheet with heat transfer were studied by Vajravelu[2001]. This work extended by Abbas and Hayat[2008] considering the stagnation slip flow and Rana and Bhargava[2012] considering the fluid as nano fluid. Mabood et al.[2015] presented the numerical solution for velocity, concentration and temperature for the flow of nanofluid past a nonlinear stretching sheet. The two-dimensional flow caused by a non linear stretching sheet with partial slip effects was discussed by Hayat et al.[2011]. The effects of chemical reaction and partial slip on the three-dimensional flow of a nano fluid impinging on an exponentially stretching surface are studied by Mahanthesh et al.[2015]. The effect of variable viscosity of viscous incompressible second grade fluid over a stretching surface in uniform magnetic field is investigated by Mahanta[2012]. Study of viscoelastic fluid flow and heat transfer over a stretching porous surface with prescribed heat and mass flux embedded in a porous medium with viscous dissipation is dicussed by Anjali and Ganga. Computational modeling of heat transfer over an unsteady stretching surface embedded in a porous medium is studied by Pal and Hiremath[2010]. Flow and heat transfer of a fluid through a porous medium over a stretching surface with internal heat generation/absorption and suction /blowing is discussed by Cortell. Effects of variable fluid properties on MHD flow discussed by Suresh Babu et al.[2018].

In most of the investigations involving heat transfer, we observed that PST (prescribe surface temperature) and PHF (prescribed wall heat flux) both not consider for non-linear stretching. In this work we consider both heat transfer cases for non-linear flow.

The objective of the present work is to analyze the development of heat and mass transfer in MHD flow over a stretching sheet with a non-uniform heat source/sink. The non-linear flow of a Newtonian liquid due to a sheet that is stretched between two blocks and heat transfer in the boundary layer flow of the stretching sheet. An exact analytical solution to the momentum equation and series solution to the energy equations in terms of Kummer's function are developed. Further, several graphs are drawn for various values of parameters like the Prandtl number, the heat source-sink parameters and Chandrasekhar number.

## II. MATHEMATICAL FORMULATIONS

We consider a steady, two-dimensional boundary layer flow of an incompressible liquid subjected to a transverse magnetic field. The liquid is at rest and the motion is effected by pulling the sheet on both ends with equal forces parallel to the sheet and a speed $u$, which varies quadratically with distance from the slit as $u = cx + dx^2$. The flow field is subjected to a transverse uniform magnetic field $H_0$ is imposed in the vertical direction y-axis. It is assumed that magnetic field is negligibly small.

The steady two-dimensional conservation of mass and the momentum boundary layer equations for the quadratic ally stretching sheet problem involving Newtonian liquids with transverse magnetic field are.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \mu_m{}^2 \sigma H_0{}^2 u, \qquad (2)$$

where $x$ and $y$ represent horizontal and transverse directions respectively, and $u$, $v$ are components of the liquid velocity in $x$ and $y$ directions, $\mu$ is the dynamic viscosity and $\upsilon$ is the kinematic viscosity.

The boundary conditions are considered as

$$\left.\begin{aligned} &u = cx + dx^n && at \quad y = 0, \\ &v = v_c + \delta x && at \quad y = 0, \\ &u = 0 && as \quad y \to \infty. \end{aligned}\right\} \qquad (3)$$

We assume $c$, $d$ and $\delta$ is quite small that facilitates the assumption of a weakly two-dimensional flow. Here n is the non linear variable if n=1 the stretching of the sheet is linear and if n=2 the sheet is stretched quadratically and flow is to be considered non-linear. In this problem we consider non-linear stretching. The constant $v_c$ represents suction velocity across the stretching sheet when $v_c$ < 0, it is blowing velocity when $v_c$ > 0 and it represents impermeability of the wall when $v_c$=0.

We now make the equations and boundary conditions dimensionless using the following definition

$$(X,Y) = \sqrt{\frac{c}{\upsilon}}(x,y), \quad (U,V,V_c) = \frac{(u,v,v_c)}{\sqrt{c\upsilon}}, \qquad (4)$$
$$\beta^* = \frac{d}{c}\sqrt{\frac{\upsilon}{c}}, \qquad \delta^* = \frac{\delta}{2c},$$

substituting dimensionless quantity (4) in the equations (1) and (2) and it takes the non-dimensional form as follows

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \qquad (5)$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} - QU, \qquad (6)$$

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where $Q=\frac{\mu_m{}^2 \sigma H_0{}^2}{c}$ is a Chandrasekhar number ($M_n = \sqrt{Q}$ is called Hartmann number),

Now we introducing the stream function $\psi(X,Y)$ as

$$U = \frac{\partial \psi}{\partial Y}, \qquad V = -\frac{\partial \psi}{\partial X}, \tag{7}$$

which satisfies the continuity equation (5), by substituting (7) in (6) we get following partial differential equation

$$\frac{\partial^3 \psi}{\partial Y^3} + \frac{\partial \psi}{\partial X}\frac{\partial^2 \psi}{\partial Y^2} - \frac{\partial \psi}{\partial Y}\frac{\partial^2 \psi}{\partial X \partial Y} - Q\frac{\partial \psi}{\partial Y} = 0. \tag{8}$$

Similarly, substituting (7) in the boundary condition (3) using dimensionless quantities (4) which obtained in the following form

$$\left.\begin{aligned} &\frac{\partial \psi}{\partial Y} = X + \beta^* X^2 && at \quad Y = 0, \\ &-\frac{\partial \psi}{\partial X} = 2V_C + 2\delta^* X && at \quad Y = 0, \\ &\frac{\partial \psi}{\partial Y} = 0 && as \quad Y \to \infty. \end{aligned}\right\} \tag{9}$$

The solution to equation (8), subject to equations (9), may be taken as

$$\psi = Xf(Y) - \beta^* X^2 f'(Y). \tag{10}$$

Substituting equation (10) into equation (8) obtain the following ordinary differential equation

$$f''' + ff'' - (f')^2 - Qf' = 0. \tag{11}$$

The boundary conditions, for solving equation (11) for dimensionless stream function *f*, can be obtained from equations (9) in the form

$$f(0) = -2V_c\,, \qquad f'(0) = 1, \qquad f'(\infty) = 0. \tag{12}$$

The solution of equations (11) subject to (12) is

$$f(Y) = \frac{1}{s}(1 - e^{-sY}) - 2V_c\,, \tag{13}$$

where 's' is given by

$$s = -V_c + \sqrt{V_c{}^2 + (1+Q)}. \tag{14}$$

Substituting equation (10) into equation (7), we can get velocity components $U$ and $V$ as

$$\mathrm{U} = \mathrm{X f'(Y)} - \beta^* \mathrm{X^2 f''(Y)}, \tag{15}$$
$$\mathrm{V} = -\mathrm{f(Y)} + 2\beta^* \mathrm{X f'(Y)}. \tag{16}$$

Having obtained the velocity distribution we discuss the heat transport in the aforementioned forced convective flow due a stretching sheet.

## III. HEAT TRANSFER ANALYSIS

The governing boundary layer heat transport equation with viscous dissipation and internal heat generation or absorption is

$$u\frac{\partial T}{\partial X} + v\frac{\partial T}{\partial Y} = \alpha^* \frac{\partial^2 T}{\partial Y^2} + \frac{Q^*}{\rho\, C_p}(T - T_\infty), \tag{17}$$

where T is the temperature of the liquid, $\alpha^*$ is the thermal diffusivity, $\mathrm{Q}^*$ uniform heat source and $\mathrm{C_p}$ specific heat at constant pressure.

The thermal boundary conditions for solving equation (17) depend on the type of heating process under consideration. We consider two different heating processes, namely (i) Prescribed Surface Temperature (PST) and (ii) Prescribed wall Heat Flux (PHF). The heat transfer analyses for these two processes are carried out in section (i) and (ii).

### (i) Prescribed Surface Temperature (PST)

The non-dimensionalized surface temperature in this case considered to be a power of X in the form

$$\begin{aligned} T = T_w = T_\infty + AX^2 \quad &at \quad Y = 0, \\ T \to T_\infty \quad &as \quad Y \to \infty, \end{aligned} \tag{18}$$

where A is a constant, $\mathrm{T_w}$ is the wall (sheet) temperature and $\mathrm{T}_\infty$ is the constant temperature far away from the sheet. We now define a non dimensional temperature

$$\Theta(\mathrm{Y}) = \frac{\mathrm{T} - \mathrm{T}_\infty}{\mathrm{T}_\mathrm{w} - \mathrm{T}_\infty}, \tag{19}$$

Where

$$\mathrm{T} - \mathrm{T}_\infty = \mathrm{A}\mathrm{X}^2\Theta(\mathrm{Y}) \quad \text{and} \quad \mathrm{T}_\mathrm{w} - \mathrm{T}_\infty = \mathrm{A}\mathrm{X}^2.$$

Substitution of Eq. (19) in the energy equation (17) leads to the following equation

$$\begin{aligned} &\Theta'' + \Pr\left(\frac{1-\exp[-sY]}{s} - 2V_c\right)\Theta' \\ &+ \Pr(\lambda - 2(\exp[-sY]))\Theta - 2 \\ &\beta^* X\left(\frac{\exp[-sY]}{s}\Theta + \exp[-sY]\Theta'\right) = 0, \end{aligned} \tag{20}$$

where prime denotes differentiation with respect to Y and the non-dimensional parameters are defined as given below:

$$\Pr = \left(\frac{\upsilon}{\alpha^*}\right) \quad \Pr\textit{andtl number.},$$

$$\lambda = \left(\frac{Q^*}{C_P \rho c}\right) \quad \textit{heat source}/\sin k \ \textit{parameters.}$$

Obviously, we get an X-independent similarity equation from equation (20)

$$\begin{aligned} &\Theta'' + \Pr\left(\frac{1-\exp[-sY]}{s} - 2V_c\right)\Theta' \\ &+ \Pr(\lambda - 2(\exp[-sY]))\Theta = 0 \end{aligned} \tag{21}$$

and

$$s\Theta' + \Theta = 0. \tag{22}$$

Equation (21) is the governing equation for the heat transfer in the flow due to the non-linearly stretching sheet and equation (22) concern the static situation.

The boundary condition in terms of $\Theta$ can be obtained from equations (18) and (19) as

$$\begin{aligned} &\Theta = 1 \quad at \quad Y = 0, \\ &\Theta \to 0 \quad as \quad Y \to \infty, \end{aligned} \tag{23}$$

equation (21) is linear in $\Theta$ and we now transform the same into a confluent hyper geometric equation by using the transformation

$$\xi = \frac{-\Pr\exp[-sY]}{s^2}, \tag{24}$$

substituting Eq.(24) into Eq.(21), we get

$$\begin{aligned} &\xi\ddot{\Theta} + \dot{\Theta}(1 - \xi - \Pr) \\ &+ \left(2 + \frac{\Pr\sigma}{\xi}\right)\Theta = 0, \end{aligned} \tag{25}$$

where over dot denotes differentiation with respect to $\xi$ and $\sigma = \lambda\Pr/\mathrm{s}^2$.

The boundary conditions in equation (25), in terms of $\xi$ translate to

$$\Theta\left(\xi = -\Pr/s^2\right) = 1 \quad \textit{and} \quad \Theta(0) = 0\,. \tag{26}$$

The solution of equation (25) satisfying the condition (26) in terms of Kummer's function is

$$\begin{aligned} &\Theta(\xi) = \left(-\xi s^2/\Pr\right)^{a+b/2} \\ &\frac{F[(a+b-4)/2,\ 1+b,\ \xi]}{F[(a+b-4)/2,\ 1+b,\ -\Pr/s^2]}, \end{aligned} \tag{27}$$

where $\mathrm{a} = \Pr$, $\mathrm{b} = \sqrt{(\Pr^2 - 4\sigma)}$.

The solution (27) can be written in terms of Y as

$$\begin{aligned} &\Theta(Y) = \exp[-(a+b)sY/2] \\ &\frac{F[(a+b-4)/2,\ 1+b, -c\exp[-sY]]}{F[(a+b-4)/2,\ 1+b,\ -c]}, \end{aligned} \tag{28}$$

where $\mathrm{c} = \Pr/\mathrm{s}^2$.

The non-dimensional wall temperature gradient derived from equation (27) is

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$$\Theta'(0) = \frac{cs}{2}(a+b-4)(1+b)^{-1}$$
$$F[(a+b-2)/2,\, 2+b,\, -c] - \tag{29}$$
$$\frac{1}{2}(a+b)\,s\,F[(a+b-4)/2, 1+b, -c] \div$$
$$F[(a+b-4)/2, 1+b, -c].$$

The non-dimensional local wall heat flux can be expressed as

$$q_w = -k\left(\frac{\partial T}{\partial Y}\right)_{Y=0} = -kAX^2\Theta'(0). \tag{30}$$

**(ii) Prescribed Heat Flux (PHF)**

The power law heat flux on the wall surface is considered to be a power of X in the form

$$-k\frac{\partial T}{\partial Y} = q_w = \overline{D}\,X^2 \quad at \quad Y=0, \qquad T \to T_\infty \quad as \quad Y \to \infty, \tag{31}$$

Where $\overline{D}$ a constant and k is is the thermal conductivity. We now define a non-dimensional temperature

$$\Phi(\mathrm{Y}) = \frac{\mathrm{T}-\mathrm{T}_\infty}{\mathrm{T_w}-\mathrm{T}_\infty}, \tag{32}$$

Where

$$T - T_\infty = \frac{\overline{D}}{k}X^2\Phi(Y) \quad and \quad T_w - T_\infty = \frac{\overline{D}}{k}X^2. \tag{33}$$

In spite of the fact that $\Phi(\mathrm{Y})$ in equation (32) is the same as $\Theta(\mathrm{Y})$ defined in equation (19) for PST case, we prefer to use a different notation for the PHF case. Substitution of equation (32) in the energy equation (17) leads to the following equation

$$\Phi'' + \Pr\left(\frac{1-\exp[-sY]}{s} - 2V_c\right)\Phi' + \Pr\left(\lambda - 2\left(\exp[-sY]\right)\right)\Phi - 2\beta^* X\left(\frac{\exp[-sY]}{s}\Phi + \exp[-sY]\Phi'\right) = 0. \tag{34}$$

Obviously, we get an X-independent similarity equation from the above equation is

$$\Phi'' + \Pr\left(\frac{1-\exp[-sY]}{s} - 2V_c\right)\Phi' + \Pr\left(\lambda - 2\left(\exp[-sY]\right)\right)\Phi = 0 \tag{35}$$

and

$$s\Theta' + \Theta = 0. \tag{36}$$

Equation (35) is the governing equation for the heat transfer in the flow due to the non-linearly stretching sheet and equation (36) concern the static situation. The boundary conditions in terms of $\Phi$ can be obtained from equation (31) and (32) as

$$\Phi'(0) = -1 \quad \text{and} \quad \Phi(\infty) = 0, \tag{37}$$

prime denotes differentiation with respect to Y and all other parameters are as defined in the PST case, but where ever A is involved in the equation of PST case it is to be replaced by $\overline{D}$ of PHF. Substituting equation (24) into (35) and (29), we get

$$\xi\ddot{\Phi} + \dot{\Phi}(1-\xi-\Pr) + \left(2 + \frac{\Pr\sigma}{\xi}\right)\Phi = 0\,, \tag{38}$$

$$\dot{\Phi}\left(-\Pr/s^2\right) = -\frac{s}{\Pr} \quad and \quad \Phi(0) = 0, \tag{39}$$

where over dot denotes differentiation with respect to $\xi$ equation (38) is a confluent hyper geometric equation and the solution for $\Phi$ satisfying equation (39) is obtained in terms of Kummer's function as

$$\Phi(\xi) = (-\xi/c)^{(a+b)/2}\,F[(a+b-4)/2,\; 1+b, \xi] \div s\left\{\begin{matrix} \frac{1}{2}(a+b)\;F[(a+b-4)/2, 1+b, -c] \\ -c\frac{1}{2(1+b)}(a+b-4) \\ F[(a+b-2)/2,\, 2+b,\, -c] \end{matrix}\right\}, \tag{40}$$

where $\mathrm{a} = \Pr$, $\mathrm{b} = \sqrt{(\Pr^2 - 4\sigma)}$ and $\mathrm{c} = \Pr/\mathrm{s}^2$.

The solution (40) can be written in terms of Y as

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$$\Phi(Y) = \exp[-sY(a+b)/2] \\ F[(a+b-4)/2, 1+b, -c\exp[-sY]] \div \\ s\left\{ \begin{array}{l} \frac{1}{2}(a+b)\ F[(a+b-4)/2, 1+b, -c] \\ -c\frac{1}{2(1+b)}(a+b-4) \\ F[(a+b-2)/2, 2+b, -c] \end{array} \right\} \quad (41)$$

The wall temperature $T_w$ is obtained from equation (33) as

$$T_w - T_\infty = \frac{\overline{D}}{k} X^2\ \Phi(0). \quad (42)$$

## IV. RESULT AND DISCUSSION

In this problem we investigate the boundary layer flow and heat transfer in a Newtonian liquid over a stretching sheet in the presence of a transverse magnetic field. Similarity solution is obtained for the velocity distribution. It is clear from equation (14) that s, which is function of the $V_c$ and Chandrasekhar number $Q$, contributes to the slope of the exponentially decreasing velocity profiles. Thus‘s’ is an important parameter in the present study. From Fig. 3 it is evident that s is an increasing function of $V_c$ and $Q$ thus implying that increasing $V_c$ and $Q$ gives us steeper gradients in the axial and transverse velocity profiles. Also from Fig. 4 and 5 it is apparent that the transverse velocity profile decays faster than the axial velocity profile for increasing $V_c$. The effect of magnetic field is to provide rigidity to the electrically conducting liquid.

The PST and PHF boundary conditions are used for solving the heat transport equation. Figs. 7 to 11 are plots of the temperature distribution for different values of Q, Pr *and* Vc.

The effect of transverse magnetic field on heat transfer is depicted in Fig. 6 and 7 for PST and PHF cases. From these plots it is observed that the transverse magnetic field contributes to the thickening of the thermal boundary layer. The resistance due to Lorentz force on the flow is responsible for enhancing the temperature.

Fig. 8 and 9 shows the effect of Prandtl number on the heat transfer in the PST and PHF cases. From these plots it is evident that large values of Prandtl number results in thinning of the thermal boundary layer. This is in contrast to the effects of other parameters on heat transfer.

The effect of $Vc$ on the heat transfer is demonstrated in Fig. 10 and 11 for PST and PHF cases. These graphs show that the parameter $V_c$ contributes to the thickening of thermal boundary layer.

On comparing the temperature distribution of the PST and PHF cases it is apparent that PST boundary condition succeeds in keeping the cooling liquid warmer than in the case when PHF boundary condition is applied. It may therefore be inferred that the PHF boundary condition is better suited for faster cooling of the stretching sheet. In contrast to the effect $V_c$ and $Q$ on $\Theta$ and $\Phi$, the effect of increasing Pr is to decrease the magnitude of $\Theta$ (Y) and $\Phi(Y)$ In other words it means that the thermal boundary layer thickness is a function of all the above parameters.

**Table 1** : Values of s for different values $Q$ *and* $V_c$.

| $V_c$ | Q | s |
|---|---|---|
| 0.1 | 0.05 | 0.929563 |
| 0.2 | 0.05 | 0.844031 |
| 0.3 | 0.05 | 0.767708 |
| 0.4 | 0.05 | 0.7 |
| 0.1 | 0.1 | 0.953565 |
| 0.2 | 0.1 | 0.867708 |
| 0.3 | 0.1 | 0.790871 |
| 0.4 | 0.1 | 0.722497 |
| 0.1 | 0.15 | 0.977033 |
| 0.2 | 0.15 | 0.890871 |
| 0.3 | 0.15 | 0.813553 |
| 0.4 | 0.15 | 0.744552 |
| 0.1 | 0.2 | 1 |
| 0.2 | 0.2 | 0.913553 |
| 0.3 | 0.2 | 0.835782 |
| 0.4 | 0.2 | 0.76619 |

***International Journal of Engineering Applied Sciences and Technology, 2020***
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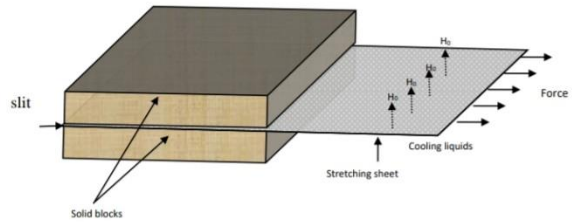


**Fig. 1:** Schematic of the 3-D stretching sheet problem.

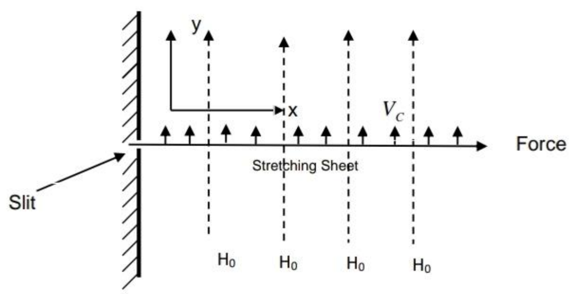


**Fig. 2:** Schematic of the 2-D stretching sheet problem.

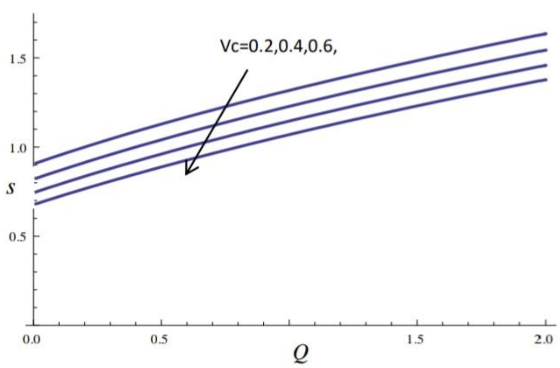


**Fig. 3:** Plot of s Vs $Q$ for different values of $V_c$.

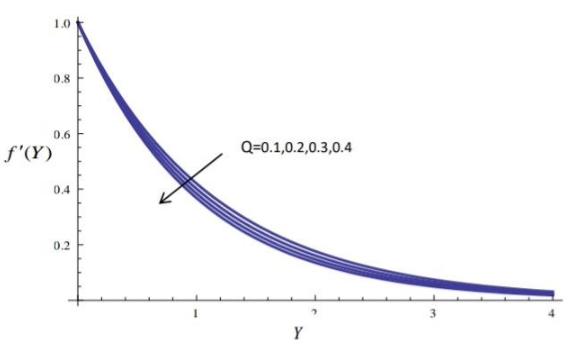


**Fig. 4:** Effect of Q on velocity profile $f'(Y)$.

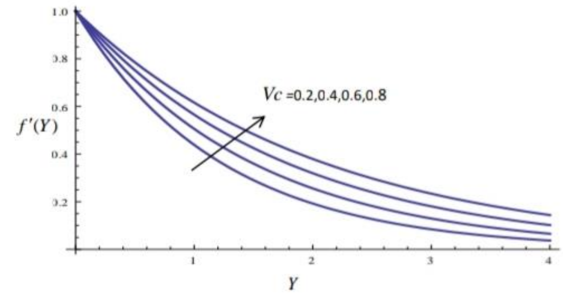


**Fig. 5:** Effect of $V_c$ on velocity profile $f'(Y)$.

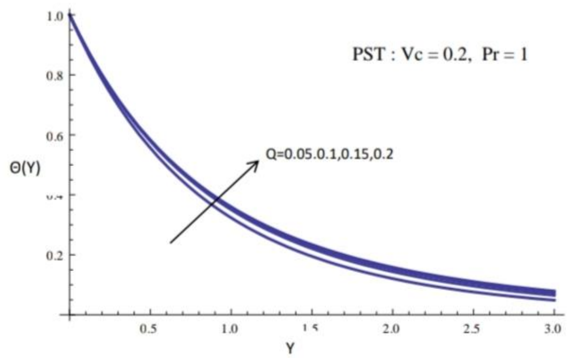


**Fig. 6:** Effect of Q on temperature profile Θ(Y) in PST case.

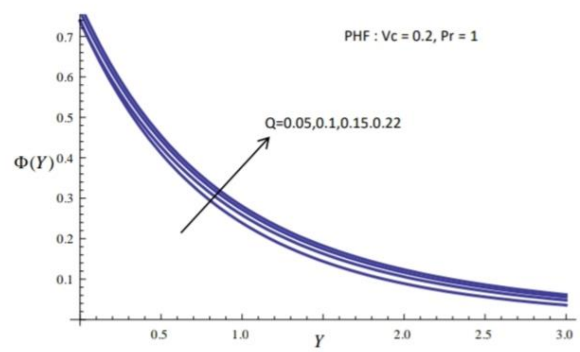


**Fig. 7:** Effect of Q on temperature profile $\Phi(Y)$ in PHF case

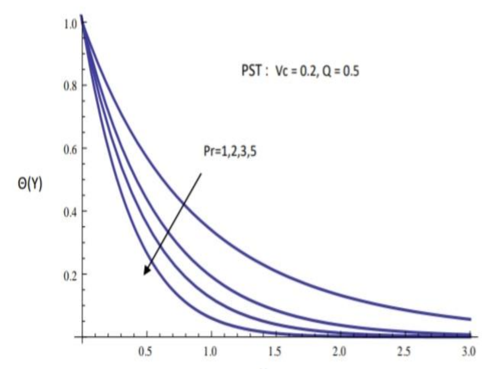


**Fig. 8:** Effect of Pr on temperature profile Θ(Y) in PST case.

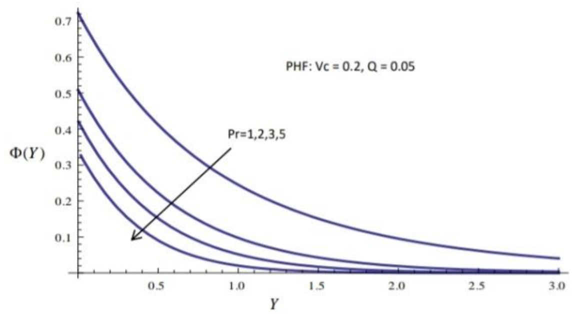


**Fig. 9:** Effect of Pr on temperature profile $\Phi(Y)$ in PHF case.

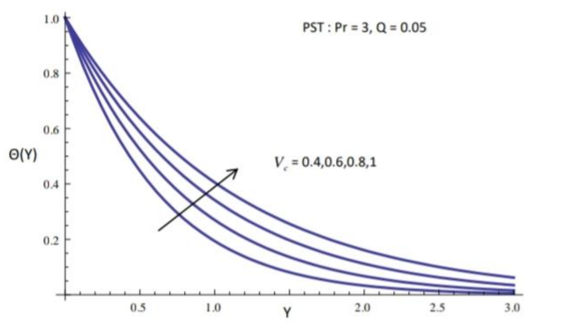


**Fig. 10:** Effect of $V_c$ on temperature profile in $\Theta(Y)$ in PST case.

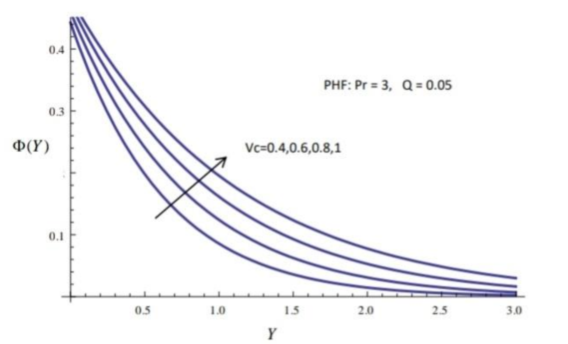


**Fig. 11:** Effect of $V_c$ on temperature profile $\Phi(Y)$ in PHF case.

## V. CONCLUSION

- The PHF boundary condition is better suited for effective cooling of the stretching sheet.
- The effect of magnetic field is to provide rigidity to the electrically conducting liquid.
- The effect of Chandrasekhar number Q is to increase the temperature distribution in the flow region in both the cases of PST and PHF hence, the strength of external magnetic field should be as mild as possible for effective cooling of the stretching sheet.
- The effect of $V_c$ is increase the temperature distribution in the flow region in both the PST and PHF cases and $Vc$ contributes to the thickening of thermal boundary layer.
- The effect of Prandtl number Pr is to decrease the thermal boundary layer thickness.

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