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AN ITERATIVE FORMULA FOR SIMULTANEOUS LOCATION OF THE ZEROS OF A POLYNOMIAL

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***Abstract*-Needless to say that the search for efficient algorithms for determining zeros of polynomials has been continually raised in many applications. In this paper we give a cubic iteration method for determining simultaneously all the zeros of a polynomial – assumed distinct – starting with 'reasonably close' initial approximations – also assumed distinct. The polynomial – in question – is expressed in its Taylor series expansion in terms of the initial approximations and their correction terms. A formula with cubic rate of convergence – based on retaining terms up to 2ndorder of the expansion in the correction terms – is derived.**

I. INTRODUCTION

The problem of determining all the zeros of a polynomial simultaneously has been considered by many, nonetheless a lot more is still being sought.

Without loss of generality, let us consider monic polynomials i.e. polynomials with 1 as leading coefficient.

Let $P(z) = \prod_{i=1 \text{ to } n} (z - w_i)$ (1)

be such a polynomial with w_i, $i = 1,2,..,n$ - assumed distinct - as its zeros and z_i

$i = 1,2,..,n$ – also assumed distinct - as their approximations.

Rewriting (1) as:

$P(z) = \prod_{i=1 \text{ to } n} (z - w_i) = \prod_{i=1 \text{ to } n} (z - z_i - \Delta_i)$ (2)

Or in expanded form , we have:-

$P(z) = \prod_{i=1 \text{ to } n} (z - z_i) - \sum_{i=1 \text{to } n} \Delta_i \prod_{k=1 \text{ to } n,\, k\neq i} (z - z_k) + \sum_{i=1 \text{to } n} \Delta_i \sum_{j=i+1 \text{to } n} \Delta_j \prod_{k=1 \text{ to } n,\, k\neq i,j} (z - z_k) + \ldots$

......(Higher Order Terms (3)

Putting $z = z_r$ in Eq. (3), we have

$P(z_r) = -\sum_{i=1 \text{to } n} \Delta_i \prod_{k=1 \text{ to } n,\, k\neq i} (z_r - z_k) + \sum_{i=1 \text{to } n} \Delta_i \sum_{j=i+1 \text{to } n} \Delta_j \prod_{k=1 \text{ to } n,\, k\neq i,j} (z_r - z_k) +$ (4)

Defining $Q(z) = \prod_{i=1 \text{ to } n} (z - z_i)$ (5)

and noting that $Q(z_r) = 0 \neq Q'(z_r) = \prod_{i=1 \text{ to } n,\, \neq r} (z_r - z_i)$, r=1,2,..,n (6)

It can be established that :

$\sum_{i=1 \text{to } n} \Delta_i \prod_{k=1 \text{ to } n,\, k\neq i} (z_r - z_k) = \Delta_r \, . \, Q'(z_r)$ (7)

II. DERIVATION OF THE METHOD

On ignoring Higher Order Terms and from Eq.s (4) to (7) , we can deduce :-

$P(z_r) + \Delta_r Q'(z_r) - \Delta_r . Q'(z_r) . \sum_{i=1 \text{to} n, i\neq r} \Delta_i/(z_r - z_i)$ (8)

$Q'(z_r) = \prod_{i=1 \text{ to } n,\, \neq r} (z_r - z_i)$ (9)

Now, truncating Eq.(8) after the 1st order term we have

$P(z_r) + \Delta_r \, . \, Q'(z_r) = 0$ (10)

Giving $\Delta_r = P(z_r) / Q'(z_r)$ (11)

hence-forth denoted by ∂_r ($\approx$ - $P(z_r) / Q'(z_r)$), the expression given by Durand Kerner, known to give quadratic convergence.

Truncating Eq. (4) after the 2ndorder term, we obtain Eq (8), which can be rearranged to give an expression for Δ_r - the theme of our method, namely:-

$$\mathbf{\Delta_r = - \; P(z_r) \; / Q'(z_r) \, [1 - \textstyle\sum_{i=1 \text{to } n,\, i \neq r} \Delta_i \,/ (z_r - z_i) \,]^{-1}} \quad (12)$$

For practical computational purposes and with ∂_r **($\approx$ - $P(z_r)$ / $Q'(z_r)$**) ,this may be approximated and rephrased as :-

$$\mathbf{\Delta_r \approx \partial_r \,/\, [1 \; - \; \textstyle\sum_{i=1 \text{to } n,\, i \neq r} \partial_i \,/ (z_r - z_i) \,]} \quad \mathbf{(13)}$$

III. CONCLUSION AND COMMENTS

The method is simple and easy to apply.

To understand and really comprehend the computational procedure and to have a feeling of the effectiveness of the method, appreciating its convergence rate, without loss of generality, it suffices to give an example of a cubic and confine our attention to finding the improvements to the initial crude approximations obtained via the first iteration cycle.

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Further better improvements can be attained via executing the pattern - repeatedly - with the new updated z's after the Δ's have been incorporated in them.

IV. .EXAMPLE

Consider the polynomial $P(z) = z^3 - z^2 - 81z + 81$, with 10, -10 and 0 as crude approximations to its zeros : 9 , -9 and 1.

z	$z_1 = 10$	$z_2 = -10$	$z_3 = 0$
$P(z_r)$	171	-209	81
$Q'(z_r)$	200	200	-100
∂_r	-.855	1.045	.81
Δ_r	-.986	.9705	.999

Updating z_r by Δ_r above r =1,2,3– in the light of the method we have

$z_1 = 9.014$ (9)	$z_2 = -9.0295$ (-9)	$z_3 = .9994$ (1) **

** numbers in () represent the actual zeros, quoted for comparison.

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