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OPTIONAL K-PHASE VACATION FOR M^X/G/1 QUEUE WITH BULK ARRIVAL AND STATE DEPENDENT RATES

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***Abstract*— In this investigation, a non-Markovian batch arrival queue with a single-server and multi-phases vacation has been studied. The multi-phases vacation means that after first phase of regular vacation, the server may take optional K-vacations. At any time when the process of service is started, it will provide service continuously until all the customers are served. But at the moment when there is no queue of customers for the service, the server goes for the first phase of regular vacation. On returning back from vacation, if no customer is found in the system, the server decides to take another vacation, which are optional. We construct Chapman Kolmogorov equations with the help of the supplementary variable technique. For analysis purpose, generating function method is employed. Various performance characteristics and special cases are established. To validate the analytical results, the numerical results and sensitivity analysis have been facilitated.**

***Keywords*— M^X/G/1 queue, Regular vacation, Optional vacation, Queue size, Supplementary variable, Neuro fuzzy, Optimal control.**

I. INTRODUCTION

Queueing systems have wide range of applications in the modeling and analysis of manufacturing process, production process, telecommunication, information technology, etc. In single vacation model, the server takes exactly one vacation when the service facility becomes empty. After coming back from vacation, if there is no customer for service, the server goes into inactive mode till the arrival of the customers. Most often in multiple vacation models, as there are no jobs to be served in the system, the server takes a vacation again and again until the customers arrive for service at service facility. This investigation deals with the study of the behavior of substantial state queueing model for M^X/G/1queue accompanied by first regular vacation and K-optional vacations. Our model includes the concept of an additional 'K' phases of vacation period for the queueing system with batch arrival and state dependent rate.

In this study, we apply optional vacation policy for non-Markovian model wherein the customers arrive in batches. Moreover, state dependent rates are also taken in to consideration. We develop Kolmogorov equations with the help of supplementary variables. Steady state distributions of several states are obtained by implementation of the probability generating function and Laplace transform. In previous study, no such information has been found by considering all assumptions namely optional vacation, bulk arrival and state dependent rates together. One of the simplest examples of such a case can be found at the doctor's clinic, wherein the patients arrive in batches. These patients are served by the doctor one by one. The doctor may go for the essential vacation followed by the optional vacations. The former can be considered as the case when the doctor needs to go for the lunch, which is essential. Furthermore, he may either go for emergency visit or for a personal work in the latter case. Thus this situation can be modeled as bulk queueing system with essential and optional vacations.

The paper is structured in a systematic way as follows: The review of literature is provided in section II, analogous for doing this investigation. The model description along with some notations are provided in section III. In section IV, mathematical model is formulated by which some performance measures are obtained, that may lead to explicit analytical results when solving the rather complicated vacation models. In next section V, some performance measures are given. Some special cases are deduced in section VI. Optimal policy is provided in section VII. Numerical results and sensitivity analysis are provided in section VIII. At last, concluding remarks related to our investigation have been given in section IX.

II. REVIEW OF LITERATURE

In recent past years, vacation models have been the subject of great interest for the queue theorists. There is no need to discuss about the increasing graph of queueing models due to its significant role in performance prediction of various congestion systems with vacation. The vacation queueing model for non-Marovian queueing system was studied by Miller (1964) for the first time in the queueing history. Boxma *et al.* (2002) developed a queueing model for an M/G/1 queue accompanied by server vacation. Chaudhury (2002 a) has discussed some aspects of a single server non-Markovian queue having two different vacation times covering multi vacation policy. Choudhury (2006) has discussed a non-Markovian queue with first regular vacation (FRV) and optional second vacation (OSV). Further, Maragathasundari and Sriniwasan (2013) and Jailaxmi and Arumuganathan (2015) studied a non-Markovian queue with multiple vacations. Bernoulli Vacation Schedule for machine interference problem with additional repairman was examined by Sharma (2016).

Batch arrival queueing systems have drawn the attention of many researchers. There are several studies on queueing system, which are dedicated to bulk queues accompanied by vacation or multi-vacations. Baba (1986) has done significant work for these models. Hur and Ahn (2005) proposed a queueing model where jobs reached at service facility in group to get service which is non-Markovian. They also used the concept of vacation and multi vacation in their study. Ke (2007) provided operating characteristic of an $M^X/G/1$ queue accompanied by different vacation policy. Manoharan and Sankara (2015) considered an $M^X/G/1$ queueing model in which after coming back from a vacation, the server may go for another optional vacation. Bulk queue with vacation and server breakdown was studied by Sharma (2016).

It may happen in several congestion situations that the server is unavailable to the customers over occasional periods of time, during which the server may perform some other auxiliary works such as fixing of tools, file organization, maintenance work on servicing secondary customers, etc. Allowing servers to take vacations makes the queueing models more realistic and flexible in studying real-world queueing situations. Krishna *et al.* (1998) proposed a multiple vacations batch arrival queue accompanied by N-policy. An analytical approach was applied by Chaudhury (2002 b) to find the consistent state distribution of queueing model at different point of time. A multi vacation bulk queueing model with obsessed arrivals was discussed by Arumuganathan and Ramaswami (2003, 2004). $M^X/G(a,b)/1$ system accompanied by more than one vacation by the incorporation of concept N-policy. Choudhury (2007) considered a queue wherein arrivals join the queue in groups with two-phase service under Bernoulli vacation schedule. Choudhury et al. (2007) proposed a queueing model where jobs arrive in group with two phases of heterogeneous service accompanied by multi vacation. Jeyakumar and Arumuganathan (2011) analyzed the steady state behavior of single server non-Markovian queue accompanied by multi vacations and control policy. A single server vacation queueing model with optional bulk service and an un-reliable server was explored by Jain et al. (2012). Maragathasundari and Karthikeyan (2016) proposed a single server non-Markovian batch arrival queue where server may go for a short or long vacations. Recently, Sharma (2017) investigated unreliable server queue with vacation interruption.

A hybrid soft computing approach pertained for performance modeling of telecommunication systems, is a Neuro-fuzzy technique. Takagi (1990) discussed many conceptional and computational aspects and also provided a survey on neuro fuzzy system. Cornelius and Leondes (1998) and Tettamanzi and Tomassini (2001) gave a description of adaptive neuro fuzzy inference systems (ANFIS). The inputs were fuzzified by using appropriate membership functions. A neural network of fuzzy system was proposed by Ciaramella et al. (2006) by using fuzzy relation. Shao et al. (2006) have presented neuro-fuzzy position control of defining tele-operation system. Ramesh and Kumara (2013) constructed the membership functions of the system characteristics of a bulk system.

III. MODEL DESCRIPTION

In the present study, we discuss steady state bulk queueing model with K-phase optional vacations. The jobs to be served, join the queue in group with different rates in compliance with the compound Poisson process; the arrival rate λ_B corresponds to the busy state, λ_R be the rate of first regular vacation (FRV) and λ_{O_i} be the corresponding arrival rates of i^{th} (i=1,2,…,K) optional vacation state. In this investigation, a single server is considered to render service exhaustively. Assume that the random variable 'U' denotes the service time having general distribution function U(x). There are two states of the server: i.e. busy state and vacation state. The idle state of the server is included in the first phase of regular vacation (FRV) of random length V_0 as the server is vacant during this state. However, after completion of FRV, the server decides to take i^{th} (i=1,2,…,K) phase of optional vacation. Now θ_i is the probability that the server takes i^{th} phase of optional vacation V_i (i=1,2,…,K), else he stays in the system with probability (1- θ_i) till the arrival of a new customer. The vacations times are i.i.d. random variables having general distribution functions $V_0(x)$ for FRV and $V_i(x)$ for i^{th} phase of optional vacation, respectively.

Now, we give some notations which are used in the model formulation.

Notations

λ_R Mean arrival rates of the customers when the server is on first regular vacation.

λ_B Mean arrival rate of the customers when the server is in busy state.

λ_{O_i} Mean arrival rates of the customers when the server is in i^th^ (i=1,2,…,K) optional vacation state.

X Random variable denoting the batch size.

E(X) Mean batch size.

c_j Pr[X=j] , j= 1,2,3,…

C(z) Generating function for the batch size X, i.e. $C(z)=\sum_{j=1}^{\infty} c_j z^j$.

$N(t)$ Number of customers in the queue at time t (excluding one being served, if any).

$U(x), V_0(x), V_i(x)$ Cumulative distribution functions (CDFs) of service time, FRV and i^{th}(i=1,2,…,K) optional vacations, respectively.

$U^*(s), V_0^*(s), V_i^*(s)$ Laplace-Steiltjes transform of CDFs of service time, FRV and i^{th}(i=1,2,…,K) optional vacations, respectively.

θ_0, θ_i Probability that the server takes FRV and i^{th}(i=1,2,…,K) vacations, respectively.

μ, γ, ω_i Mean service rate, FRV rate and i^{th}(i=1,2,…,K) optional vacation rate, respectively.

E[U], E[U^j] 1^{st} and j^{th} (j>1) moments of service time.

E[V], E[V^j] 1^{st} and j^{th} (j>1) moments of vacation time, *l*=0,1,2…,,K

Let

$V_O(t)$ Elapsed service time of first regular vacation (FRV) at time t.

$V_i(t)$ Elapsed service time of i^{th} (i=1,1,…K) regular vacation (FRV) at time t.

U(t) Elapsed service time of customers during busy state at time t.

We define the hazard rate functions as follows:

$$\mu(x)=\frac{dU(x)}{[1-U(x)]}; \quad \gamma(x)=\frac{dV_0(x)}{[1-V_0(x)]}; \quad \omega_i(x)=\frac{dV_i(x)}{[1-V_i(x)]},$$

i=1,2,…,K

IV. MODEL FORMULATION

In this section, using the supplementary variable as elapsed service time and elapsed vacation times, we develop a queueing system with the help of steady-state differential equation. Let us define the random variable $\xi(t)$ which denotes the server's state as follows:

$$\xi(t)=\begin{cases} 0, & \text{if the server is on FRV at time 't'} \\ i, & \text{if the server is on optional } i^{th} \text{ vacation} \\ & \text{at time 't'} \;;\; i=1,2,\ldots,K \\ K+1, & \text{if the server is busy at time 't'}. \end{cases}$$

Consider bivariate Markov process {N(t), δ(t); t>0}, where δ(t) is a supplementary variables defined as

$$\delta(t)=\begin{cases} V_i(t), & \text{when } i=0,1,\ldots,K \\ U(t) \end{cases}$$

Now the limiting probabilities are defined as follows:

$$Q_{i,n}(x)dx=\lim_{t\to\infty}\Pr\{N(t)=n, \xi(t)=i, \delta(t)=V_i(t); x<V_i(t)\le x+dx\};$$

$$n\ge 0, x>0 \text{ and } i=0,1,2,\ldots,K$$

$$P_n(x)dx=\lim_{t\to\infty}\Pr\left\{\begin{matrix} N(t)=n, \xi(t)=K+1, \delta(t)=U(t); \\ x<U(t)\le x+dx \end{matrix}\right\}; n\ge 0, x>0$$

(A) The governing equations

Now, we construct the Kolmogorov forward equations at stationary point of time by using supplementary variables to examine the limiting form of the queueing process as follows:

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$$\frac{d}{dx}P_n(x)+[\lambda_B+\mu(x)]P_n(x)=\sum_{j=1}^{n}\lambda_B c_j P_{n-j}(x)\ ; \quad n\geq 0 \tag{1}$$

$$\frac{d}{dx}Q_{0,n}(x)+[\lambda_R+\gamma(x)]Q_{0,n}(x)=\sum_{j=1}^{n}\lambda_R c_j Q_{0,n-j}(x)\ ; \quad n\geq 0 \tag{2}$$

$$\frac{d}{dx}Q_{i,n}(x)+[\lambda_{O_i}+\omega_i(x)]Q_{i,n}(x)=\sum_{j=1}^{n}\lambda_{O_i} c_j Q_{i,n-j}(x)\ ; \quad n\geq 0, \quad \text{i=1,2,…,K} \tag{3}$$

$$\lambda_R Q_{0,0}=\int_0^\infty \mu(x)P_0(x)dx+(1-\theta_0)\int_0^\infty \gamma(x)Q_{0,0}(x)dx+(1-\theta_i)\int_0^\infty \omega_i(x)Q_{i,0}(x)dx+\int_0^\infty \omega_K(x)Q_{K,0}(x)dx; \quad i=1,2,3,\ldots,K \tag{4}$$

Here $Q_{0,0}$ is the probability of the server being on FRV and no customer present in the system. Here we have some boundary conditions to obtain a solution of the above equations (1)-(4) at x=0.

$$Q_{0,0}(0)=\lambda_R Q_{0,0} \tag{5}$$

$$Q_{0,n}(0)=0; \qquad n\geq 1 \tag{6}$$

$$Q_{1,n}(0)=\theta_0\int_0^\infty \gamma(x)Q_{0,n}(x)dx; \quad n\geq 0 \tag{7}$$

$$Q_{i,n}(0)=\theta_{i-1}\int_0^\infty \omega_{i-1}(x)Q_{i-1,n}(x)dx; \quad n\geq 0; \ \text{i=2,3,…,K} \tag{8}$$

The normalization condition is given by

$$P(1)+\sum_{i=0}^{K}Q_i(1)=1 \tag{10}$$

Define Laplace-Stieltjes transform of a function F(t) as $\overline{F}(s)=\int_0^\infty e^{-st}dF(t)$.

To solve the system of equations (1)-(10), we define the following generating functions:

$$P(x;z)=\sum_{n=0}^{\infty}z^nP_n(x); \qquad x\geq 0,\ |z|\leq 1 \tag{11}$$

$$Q_i(x;z)=\sum_{n=0}^{\infty}z^nQ_{i,n}(x); \qquad x\geq 0,\ |z|\leq 1,\ \text{i=2,3,…} \tag{12}$$

For brevity, we shall use the following notations:

$\Lambda_{O_i}(z)=\lambda_{O_i}(1-C(z))$; (i=1,2,…,K), $\Lambda_R(z)=\lambda_R(1-C(z))$, $\Lambda_B(z)=\lambda_B(1-C(z))$

B. Mathematical analysis

In this section we have done some mathematical analysis.

Theorem 1: The joint generating function of stationary distributions of the process $\{N(t), t\geq 0\}$ is given by

$$P(x;z)=\frac{\left\{\lambda_R Q_{0,0}\left[1-\left\{\overline{\theta_0}+\overline{\theta_1}\theta_0V_1^*[\Lambda_{O_1}(z)]+\overline{\theta_i}\theta_0\left(\prod_{j=1}^{i-1}\theta_jV_j^*[\Lambda_{O_j}(z)]\right)V_i^*[\Lambda_{O_i}(z)]+V_K^*[\Lambda_{O_K}(z)]V_{K-1}^*[\Lambda_{O_{K-1}}(z)]\left(\prod_{j=1}^{K-2}\theta_jV_j^*[\Lambda_{O_j}(z)]\right)\theta_{K-1}\theta_0\right\}V_0^*[\Lambda_R(z)]\right]\right\}e^{-\Lambda_B(z)x}[1-U(x)]}{U^*[\Lambda_B(z)]-z}, \quad \text{i=2,3,…,K} \tag{13}$$

$$Q_0(x;z)=\lambda_R Q_{0,0}\,e^{-\Lambda_R(z)x}[1-V_0(x)] \tag{14}$$

$$Q_1(x;z)=\theta_0\lambda_R Q_{0,0}V_0^*[\Lambda_R(z)]e^{-\Lambda_{O_1}(z)x}[1-V_1(x)] \tag{15}$$

$$Q_i(x;z)=\theta_0\lambda_R Q_{0,0}V_0^*[\Lambda_R(z)]e^{-\Lambda_{O_i}(z)x}[1-V_i(x)]\left(\prod_{j=1}^{i-1}\theta_jV_j^*[\Lambda_{O_j}(z)]\right), \quad \text{i=2,3,…,K-1} \tag{16}$$

$$Q_K(x;z)=\theta_0\theta_{K-1}\lambda_R Q_{0,0}V_0^*[\Lambda_R(z)]e^{-\Lambda_{O_K}(z)x}[1-V_K(x)]\left(\prod_{j=1}^{K-2}\theta_jV_j^*[\Lambda_{O_j}(z)]\right)V_{K-1}^*[\Lambda_{O_{K-1}}(z)] \tag{17}$$

Proof: For solving eq. (1), we convert it into the form of generating function by implementation of suitable operations.

$$\frac{d}{dx}P(x;z)+[\lambda_B+\mu(x)-\lambda_BC(z)]P(x;z)=0 \tag{18}$$

Similarly from eqs (2)-(12), we get

$$\frac{d}{dx}Q_0(x;z)+[\lambda_R+\gamma(x)-\lambda_RC(z)]Q_0(x;z)=0 \tag{19}$$

$$\frac{d}{dx}Q_i(x;z)+[\lambda_{O_i}+\omega_i(x)-\lambda_{O_i}C(z)]Q_i(x;z)=0, \quad \text{i=1,2,…,K-1} \tag{20}$$

$$\frac{d}{dx}Q_K(x;z)+[\lambda_{OK}+\omega_K(x)-\lambda_{OK}C(z)]Q_K(x;z)=0 \quad (21)$$

On solving eq. (9) and using eq. (4), we get

$$Q_0(0;z)=\lambda_R Q_{0,0} \quad (22)$$

$$Q_1(0;z)=\theta_0\int_0^\infty Q_0(x;z)\gamma(x)dx \quad (23)$$

$$Q_i(0;z)=\theta_{i-1}\int_0^\infty Q_{i-1}(x;z)\omega_{i-1}(x)dx\text{, i=2,3,…,K-1} \quad (24)$$

$$Q_K(0;z)=\theta_{K-1}\int_0^\infty Q_{K-1}(x;z)\omega_{K-1}(x)dx \quad (25)$$

Solving eqs (18)-(24), we obtain

$$P(x;z)=P(0;z)e^{-\Lambda B(z)x}[1-U(x)] \quad (26)$$

$$Q_0(x;z)=Q_0(0;z)e^{-\Lambda R(z)x}[1-V_0(x)] \quad (27)$$

$$Q_1(x;z)=Q_1(0;z)e^{-\Lambda O_1(z)x}[1-V_1(x)] \quad (28)$$

$$Q_i(x;z)=Q_i(0;z)e^{-\Lambda O_i(z)x}[1-V_i(x)]\text{, i=2,3,…,K-1} \quad (29)$$

$$Q_K(x;z)=Q_K(0;z)e^{-\Lambda O_K(z)x}[1-V_K(x)] \quad (30)$$

Also,

$$Q_1(0;z)=\theta_0\lambda_R Q_{0,0}V_0^*(\Lambda_R) \quad (31)$$

$$Q_i(0;z)=\theta_0\lambda_R Q_{0,0}V_0^*(\Lambda_R)\prod_{j=1}^{i-1}\theta_j V_j^*[\Lambda_{o_j}(z)]\text{, i=2,3,…,K-1} \quad (32)$$

$$Q_K(0;z)=\theta_{K-1}Q_{K-1}(0;z)V_{K-1}^*(\Lambda_R)[\Lambda_{OK-1}(z)] \quad (33)$$

$$zP(0;z)=-\lambda_R Q_{0,0}+\int_0^\infty P(x;z)\mu(x)dx+\overline{\theta_0}\int_0^\infty Q_0(x;z)\gamma(x)dx+\sum_{i=1}^{K-1}\overline{\theta_i}\int_0^\infty Q_i(x;z)\omega_i(x)dx+\int_0^\infty Q_K(x;z)\omega_K(x)dx \quad (34)$$

Using eqs (26)-(29) into eq. (34), we get

$$P(0;z)=\frac{\left\{\overline{\theta_0}Q_0(0;z)V_0^*[\Lambda_R(z)]+\overline{\theta_1}Q_1(0;z)V_1^*[\Lambda_{O_1}(z)]+\sum_{i=2}^{K-1}\overline{\theta_i}Q_i(0;z)V_i^*[\Lambda_{O_i}(z)]+Q_K(0;z)V_K^*[\Lambda_{OK}(z)]-\lambda_R Q_{0,0}\right\}}{z-U^*[\Lambda_B(z)]} \quad (35)$$

Using eqs (22), (31), (32), (34) in eq. (35), we obtain

$$P(0;z)=\frac{\left\{\lambda_R Q_{0,0}\left[1-\left\{\overline{\theta_0}+\overline{\theta_1}\theta_0V_1^*[\Lambda_{O_1}(z)]+\overline{\theta_i}\theta_0\left(\prod_{j=1}^{i-1}\theta_jV_j^*[\Lambda_{O_j}(z)]\right)V_i^*[\Lambda_{O_i}(z)]+V_K^*[\Lambda_{OK}(z)]V_{K-1}^*[\Lambda_{OK-1}(z)]\left(\prod_{j=1}^{K-2}\theta_jV_j^*[\Lambda_{O_j}(z)]\right)\theta_{K-1}\theta_0\right\}V_0^*[\Lambda_R(z)]\right]\right\}}{U^*[\Lambda_B(z)]-z},$$

i=2,3,…,K-1 (36)

By using eqs (26) and (36), we get eq (13).

Proceeding in similar manner from eqs (22) & (27) and (24) & (29), we get eqs (14)-(16).

Theorem 2: The marginal generating functions when (i) the server is in busy state, (ii) the server is on first regular and i[th] (i=1,2,…K) optional vacations, are respectively, given by

$$P(z)=\frac{\left\{\lambda_R Q_{0,0}V_0^*[\Lambda_R(z)]\left[1-\{\overline{\theta_0}+\overline{\theta_1}\theta_0V_1^*[\Lambda_{O_1}(z)]+\overline{\theta_i}\theta_0\prod_{j=1}^{i-1}\theta_jV_j^*[\Lambda_{O_j}(z)]V_i^*[\Lambda_{O_i}(z)]+{}_K^*[\Lambda_{OK}(z)]VV_{K-1}^*[\Lambda_{OK-1}(z)]\prod_{j=1}^{K-2}\theta_jV_j^*[\Lambda_{O_j}(z)]\theta_{K-1}\theta_0\}V_0^*[\Lambda_R(z)]\right]\{1-U^*[\Lambda_B(z)]\}\right\}}{\Lambda_B(z)\{U^*[\Lambda_B(z)-z]\}} \quad (37)$$

$$Q_0(z)=Q_{0,0}\frac{[1-V_0^*[\Lambda_R(z)]]}{1-C(z)} \quad (38)$$

$$Q_1(z)=Q_{0,0}\theta_0\lambda_R V_0^*[\Lambda_R(z)]\left[\frac{1-V_1^*[\Lambda_{O_1}(z)]}{\Lambda_{O_1}(z)}\right] \quad (39)$$

$$Q_i(z)=Q_{0,0}\theta_0\lambda_R V_0^*[\Lambda_R(z)]\left(\prod_{j=1}^{i-1}\theta_jV_j^*[\Lambda_{O_j}(z)]\right)\left[\frac{1-V_i^*[\Lambda_{O_i}(z)]}{\Lambda_{O_i}(z)}\right],$$

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i=2,3,…,K-1 (40)

$$Q_K(z) = Q_{0,0}\theta_{K-1}\theta_0\lambda_R\left(\prod_{j=1}^{K-2}\theta_j V_j^*[\Lambda_{Oj}(z)]\right)\left[\frac{1-V_K^*[\Lambda_{OK}(z)]}{\Lambda_{OK}[z]}\right] V_0^*[\Lambda_R(z)]V_{K-1}^*[\Lambda_{OK-1}(z)] \quad (41)$$

Proof: The results given in equations (22)-(27) are obtained by using $P(z)=\int_0^\infty P(x;z)dx$

and $Q_i(z)=\int_0^\infty Q_i(x;z)dx$, (i=0,1,2…K).

Theorem 3: At time 't'the probability generating function of the queueing system is provided by

$$\phi(z) = P(z)\sum_{i=0}^{K} Q_i(z) \quad (42)$$

Proof: Using marginal generating functions obtained in eqs (37)-(40), we obtain eq. (42).

V. Performance Measures

In this section the probabilities for different system states and some of the performance indices with the help of previous result, are established.

(i) The probability that the server is under busy state.

$P_B = \lim_{z\to 1} P(z)$

$$= \frac{\left[E[U]\left\{\left(\overline{\theta_0}+\overline{\theta_1}\theta_0+\overline{\theta_i}\theta_0\prod_{j=1}^{i-1}\theta_j+\theta_{K-1}\theta_0\prod_{j=1}^{K-2}\theta_j\right)\delta_R + \overline{\theta_1}\theta_0\delta_1+\overline{\theta_i}\theta_0\left(\prod_{j=1}^{i-1}\theta_j\delta_i+\prod_{j=1}^{i-1}\theta_j\delta_j\right) + \theta_{K-1}\theta_0\left(\prod_{j=1}^{K-2}\theta_j\delta_j+\prod_{j=1}^{K-2}\theta_j(\delta_{K-1}+\delta_K)\right)\right\}\right]\lambda_R Q_{0,0}}{1-\rho}$$

i=2,3,…,K (43)

(ii) The probability that the server takes first regular vacation (FRV).

$$P_{V_0} = \lim_{z\to 1} Q_0(z) = \lambda_R Q_{0,0}E[V_0] \quad (44)$$

(iii) The probability that the server goes for optional 1st phase of vacation.

$$P_{V_1} = \lim_{z\to 1} Q_1(z) = \lambda_R Q_{0,0}\theta_0 E[V_1] \quad (45)$$

(iv) The probability that the server goes for optional ith phase of vacation.

$$P_{V_i} = \lim_{z\to 1} Q_i(z) = \lambda_R Q_{0,0}E[V_i]\prod_{j=0}^{i-1}\theta_j \; ; \quad \text{i=2,3,…,K} \quad (46)$$

(v) The probability that the server goes for optional Kth phase of vacation.

$$P_{V_K} = \lim_{z\to 1} Q_K(z) = \lambda_R Q_{0,0}E[V_K]\prod_{j=0}^{K-1}\theta_j \quad (47)$$

where, $\delta_R = E[V_0]\lambda_R C'(1)$; $\delta_i = E[V_i]\lambda_{O_i}C'(1)$, (i=1,2,…,K); $\rho = \lambda_B C'(1)E[U]$

Using normalization condition given in eq. (10), the unknown constant $Q_{0,0}$ can be obtained as

$$Q_{0,0} = \frac{1-\rho}{\lambda_R\left[\left(E[V_0]+\theta_0E[V_1]+\theta_0E[V_i]\prod_{j=1}^{i-1}\theta_j\right)(1-\rho) + E[U]\left[\left\{\overline{\theta_0}+\overline{\theta_1}\theta_0+\overline{\theta_i}\theta_0\prod_{j=1}^{i-1}\theta_j+\theta_{K-1}\theta_0\prod_{j=1}^{K-2}\theta_j\right\}\delta_R + \overline{\theta_1}\theta_0\delta_1+\overline{\theta_i}\theta_0\left(\prod_{j=1}^{i-1}\theta_j\delta_i+\prod_{j=1}^{i-1}\theta_j\delta_j\right) + \theta_{K-1}\theta_0\left\{\prod_{j=1}^{K-2}\theta_j\delta_j+\prod_{j=1}^{K-2}\theta_j(\delta_{K-1}+\delta_K)\right\}\right]\right]},$$

i=2,3,…,K-1 (48)

(vi) The expected queue size distribution is obtained as

$$E(S) = \lim_{z\to 1}\frac{d}{dz}\phi(z) = \sum_{m=0}^{4}\phi_m(1) \quad (49)$$

where

$$\phi_0(1) = \frac{\begin{gathered}\lambda_R Q_{0,0}\Big[\left\{-2a_{11}a_{12}E[U]-(a_{11})^3E[U^2]+a_{12}\right\}\Big[2a_{11}E[U]\Big\{a_1a_{13}E[V_0] + \overline{\theta_1}\theta_0\delta_1+\overline{\theta_i}\theta_0\prod_{j=1}^{i-1}\theta_j\delta_i+\overline{\theta_i}\theta_0\prod_{j=1}^{i-1}\theta_j\delta_j+\theta_{K-1}\theta_0\prod_{j=1}^{K-2}\theta_j\delta_j+\theta_{K-1}\theta_0 \\ \prod_{j=1}^{K-2}\theta_j(\delta_{K-1}+\delta_K)\Big\}+\{a_{12}E[U]-(a_{11})^2E[U^2]\}(1-a_1)\Big]+a_{11}E[U](1-a_1) \\ \left[2\left\{-2a_{11}a_{14}E[U]-(a_{11})^4E[U^3]+a_{14}\right\}-9(a_{11})^2a_{12}E[U^2]-3(a_{12})^4E[U]\right]\Big]\end{gathered}}{4(a_{11})^2(a_{11}E[U]-1)^2}$$

i=2,3,…,K-1 (50)

$$\phi_1(1) = \frac{Q_{0,0}\lambda_R^2 C'(1)E[V_0^2]}{2} \quad (51)$$

$$\phi_2(1)=\frac{Q_{0,0}\theta_0\lambda_R C'(1)\left\{\lambda_{O_1}E[V_1^2]+2\lambda_R E[V_0]E[V_1]\right\}}{2} \tag{52}$$

$$\phi_3(1)=\frac{\sum_{i=2}^{K-1}\left[\prod_{j=1}^{i-1}\begin{matrix}\theta_j C'(1)\left\{\lambda_{O_i}E[V_i^2]+2\lambda_R E[V_i]E[V_0]\right\} \\ +2E[V_i]\prod_{j=1}^{i-1}\theta_j\delta_j\end{matrix}\right]Q_{0,0}\theta_0\lambda_R}{2},$$

i=2,3,…,K-1 (53)

$$\phi_4(1)=\frac{Q_{0,0}\theta_0\lambda_R\left[\begin{matrix}\prod_{j=1}^{K-2}\theta_j\left\{2E[V_0]\delta_K+\lambda_{O_K}C'(1)E[V_K^2]+2E[V_K]\delta_{K-1}\right\} \\ +2E[V_K]\prod_{j=1}^{K-2}\theta_j\delta_j\end{matrix}\right]}{2}$$

(54)

For brevity of notations, we have used

$a_1=\overline{\theta_0}+\overline{\theta_1}\theta_0+\overline{\theta_i}\prod_{j=0}^{i-1}\theta_j+\prod_{j=0}^{K-1}\theta_j$, i=2,3,…,K-1

$a_{11}=\lambda_B C'(1)$, $a_{12}=\lambda_B C''(1)$, $a_{13}=\lambda_R C'(1)$,
$a_{14}=\lambda_B C'''(1)$

VI. SPECIAL CASES

It is interesting to deduce analytical results in some particular cases by setting appropriate parameters as follows:

Case I: $M^X/G/1$ queueing model with first regular vacation (FRV) and optional second vacation (OSV) policy.

Substituting $\lambda_i=\lambda$, $\theta_i=\theta$, K=1, eq. (42) yields

$$\phi(z)=\frac{(1-\rho)\left[1-\left\{\overline{\theta}+\theta V_1^*[\lambda(1-C(z))]\right\}V_0^*[\lambda(1-C(z))]\right](1-z)}{(1-C(z))\left\{U^*[\lambda_B(1-C(z))]-z\right\}\left\{\lambda E[V_0]+\lambda\theta E[V_1]\right\}} \tag{55}$$

In particular, when the jobs reach one by one, so that C(z)=z, eq. (55) becomes

$$\phi(z)=\frac{(1-\rho)[1-\left\{\overline{\theta}+\theta V_1^*(\lambda-\lambda z)\right\}V_0^*(\lambda-\lambda z)]}{\left(U^*(\lambda-\lambda z)-z\right)\left[\lambda E[V_0]+\lambda\theta E[V_1]\right]} \tag{56}$$

Also, we get

$$E(S)=\frac{(1-\rho)\lambda\,[E(V_0^2)+2\theta E(V_0)E(V_1)+\theta E(V_1^2)]}{2[E(V_0)+\theta E(V_1)]} \tag{57}$$

which agrees with the results of Choudhury (2006).

Case II: State dependent batch arrival queue for a non-Markovian system and OSV policy.

Substituting $\theta_i=\theta$, K=1, eq. (42) provides

$$\phi(z)=\frac{\begin{matrix}\left\{(1-\rho)\left[\lambda_{O_1}\lambda_R V_0^*[\lambda_R(1-C(z))]\left(1-U^*[\lambda_B(1-C(z))]\right)\right.\right. \\ \left\{1-\left(\overline{\theta}+\theta V_1^*[\lambda_{O_1}(1-C(z))]\right)V_0^*[\lambda_R(1-C(z))]\right\} \\ +\left(1-V_0^*[\lambda_R(1-C(z))]\right)\lambda_{O_1}\lambda_B\left(U^*[\lambda_B(1-C(z))]-z\right) \\ \left.\left.+\theta\lambda_B\lambda_R V_0^*[\lambda_R(1-C(z))]\begin{matrix}\left(U^*[\lambda_B(1-C(z))]-z\right) \\ \left(1-V_1^*[\lambda_{O_1}(1-C(z))]\right)\end{matrix}\right]\right\}\end{matrix}}{\begin{matrix}\left\{\left[E[U]\left\{\lambda_R C'(1)E[V_0]+\lambda_{O_1}C'(1)\theta E[V_1]\right\}+(1-\rho)\right]\right. \\ \left.\left\{E[V_0]+\theta E[V_1]\right\}(1-C(z))\lambda_{O_1}\lambda_B\lambda_R\left(U^*[\lambda_B(1-C(z))]-z\right)\right\}\end{matrix}}$$

(58)

When the customers arrive singly, by substituting, C(z)=z in eq. (58), we obtain

$$\phi(z)=\frac{\begin{matrix}\left\{(1-\rho)\left[\lambda_{O_1}\lambda_R V_0^*[\lambda_R(1-z)\left(1-U^*[\lambda_B(1-z)]\right)\right.\right. \\ \left\{1-\left(\overline{\theta}+\theta V_1^*[\lambda_{O_1}(1-z)]\right)V_0^*[\lambda_R(1-z)]\right\}+\left(1-V_0^*[\lambda_R(1-z)]\right)\lambda_{O_1} \\ \lambda_B\left(U^*[\lambda_B(1-z)]-z\right)+\theta\lambda_B\lambda_R V_0^*[\lambda_R(1-z)] \\ \left.\left.\left(U^*[\lambda_B(1-z)]-z\right)\left(1-V_1^*[\lambda_{O_1}(1-z)]\right)\right]\right\}\end{matrix}}{\begin{matrix}(1-z)\lambda_{O_1}\lambda_B\lambda_R\left(U^*[\lambda_B(1-z)]-z\right) \\ \left[\begin{matrix}E[U]\left\{\lambda_R E[V_0]+\lambda_{O_1}\theta E[V_1]\right\} \\ +(1-\rho)\left\{E[V_0]+\theta E[V_1]\right\}\end{matrix}\right]\end{matrix}}$$

(59)

Case (III): $M^X/G/1$ queue accompanied by only one vacation.

Substituting θ=0 in case (i), we have

$$\phi(z)=\frac{(1-\rho)\left[1-V_0^*[\lambda(1-C(z))]\right](1-z)}{(1-C(z))\left\{U^*[\lambda_B(1-C(z))]-z\right\}\lambda E[V_0]} \tag{60}$$

In particular case of single arrivals, by substituting C(z)=z, eq. (60) reduces to

$$\phi(z)=\frac{(1-\rho)\left[1-V_0^*[\lambda(1-z)]\right]}{\left\{U^*[\lambda_B(1-z)]-z\right\}\lambda E[V_0]} \tag{61}$$

Also we get

$$E[S]=\frac{(1-\rho)\lambda\,E[V_0^2]}{2E[V_0]} \tag{62}$$

VII. OPTIMAL POLICY

To develop the cost function, by the use of performance indices obtained above, the total cost TC(K) is given by

$$TC(K)=C_S+C_BP_B+C_{V0}P_{V0}+C_{V1}P_{V1}+\sum_{i=2}^{K-1}C_{V_i}P_{V_i}+C_{VK}P_{VK}+C_hE(S) \quad (63)$$

where

C_S : Start-up cost per unit time for turning the server on
C_B : Cost incurred per unit time for keeping the server busy
C_{V0} : FRV cost per unit time incurred on the server
C_{Vi} : Cost per unit time due to $i^{th}(1 \le i \le K)$ vacation
C_h : Holding cost per unit time per customer present in the system

Now in order to obtain the optimal values (K*), the direct search method (i.e. heuristic approach) is employed.

VIII. NUMERICAL RESULTS AND SENSITIVITY ANALYSIS

The numerical results for model developed are validated through numerical experiment. To examine the effect of different parameters on queue length, we elaborate a sensitivity analysis by varying input parameters λ, ω, μ and r. MATLAB is used for programming purpose.

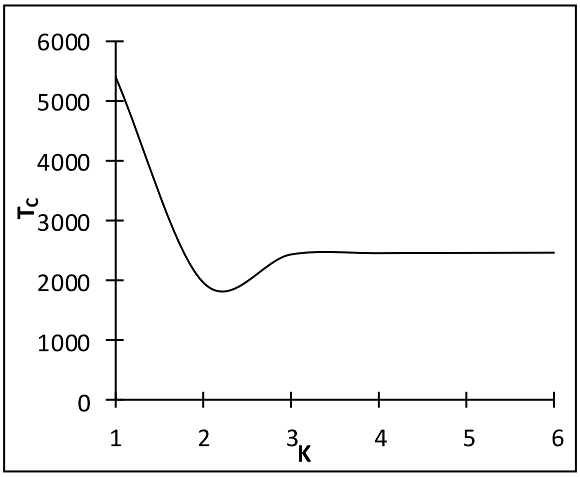
Figs 1(a)-1(d) show the results for the optimal total cost. For illustration, we assume the distribution of batch size as geometric. The vacation and service times are supposed to be exponentially distributed. Figs 1(a)-1(b) show the profile for expected total cost of the system for cost sets I and II, respectively. By setting parameters as $\lambda_{O_1}=0.11\lambda$, $\lambda_B=0.18\lambda$, $\lambda_R=0.1\lambda$, $\lambda_{OK}=\lambda$, μ=9, ω=0.2, θ=0.5, λ=7 and E(X)=3, the minimum costs are noted to be Rs 1968.85 and Rs 1562.73 for sets I and II, respectively which are achieved at K=2. Similarly, for figs. 1(c)-1(d), we fix default parameters as $\lambda_{O_1}=0.11\lambda$, $\lambda_B=0.18\lambda$, $\lambda_R=0.11\lambda$, $\lambda_{OK}=\lambda$, μ=16.8, ω=0.2, θ=0.5, λ=8.5 and E(X)=3 and obtain the optimal cost as Rs 743.15 and Rs 1093.75 at K=3 for cost sets III and IV, respectively.

Figs 2-4 depict the queue length E(L) for different values of batch size E(X) and probability of vacation θ by fixing default parameters $\lambda_{O_1}=0.1\lambda, \lambda_B=0.01\lambda, \lambda_R=0.6\lambda, \lambda_{OK}=0.7\lambda$, μ=0.6, ω=2, θ=0.5, λ=0.02 and E(X)=3. It is seen from fig. 2 that by increasing ω, the queue length increases whereas fig 5 and 6 shows the depletion in queue length with the increased service rate μ. From figs 2-4, we also examine the increasing pattern of E(L) with the increase in E(X) and θ, which is in agreement with physical situations.

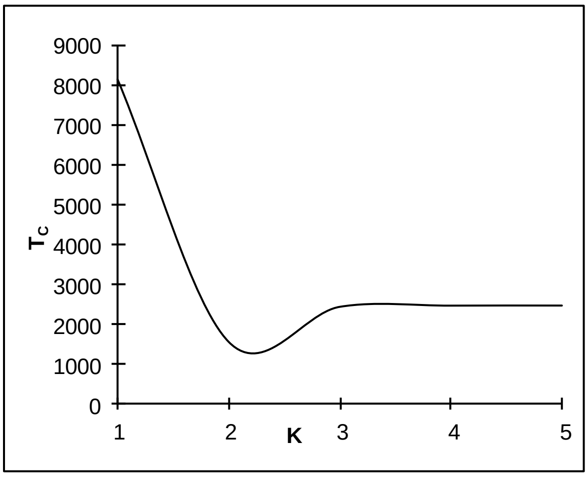
In fig 5, for the fix parameters $\lambda_{O_1}=0.01\lambda$, $\lambda_B=0.1\lambda$, $\lambda_R=0.7\lambda$, $\lambda_{OK}=0.18\lambda$, μ=2, ω=0.06,θ=0.5 and λ=0.5, the variations in the average queue length with respect to batch size E(X) and Erlangian parameter (r) have been shown through bar graph. An increasing trend of E[L] is observed as the value of r and E[X] increase.

In figs 6-7, ANFIS results for the average queue length E(L) are displayed; the computational work is done using the fuzzy toolbox of the MATLAB package. The performance measures for E(L) are obtained by varying Erlangian distribution parameter 'r'. The input parameters are treated as the linguistic variables in the context of the fuzzy systems. While building the respective ANFIS networks, these parameters are taken as the input values. The Gaussian function is used for describing the membership functions (mfs) for input parameters. For all the illustrations, the ANFIS networks are trained for 10 epochs.

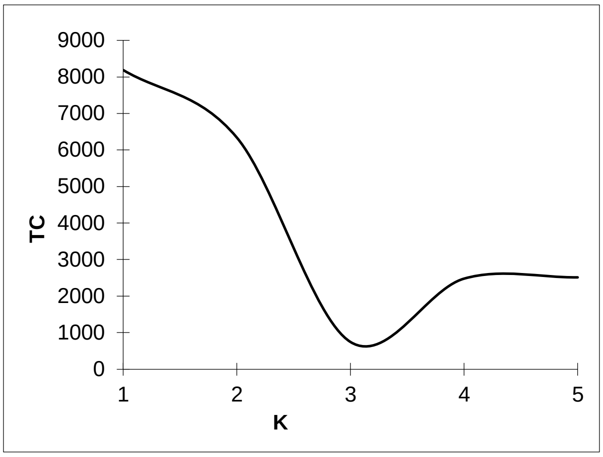
In figs 6(b)-7(b), we plot E(L) using analytical results by continuous line and the neuro fuzzy results by dotted lines. For figs 6(a-b), we set $\lambda_{O_1}=0.1\lambda$, $\lambda_B=0.1\lambda$, $\lambda_R=0.9\lambda$, $\lambda_{OK}=0.18\lambda$, μ=0.6, ω=0.2, θ=0.5, E(X)=3 and for figs 7(a-b), we set parameters as $\lambda_{O_1}=0.01\lambda$, $\lambda_B=0.1\lambda$, $\lambda_R=0.7\lambda$, $\lambda_{OK}=0.18\lambda$, λ=0.5 and E(X)=1, μ=0.6, ω=0.06 . From figs 6(b) and 7(b), it can be seen that as λ and θ increase, the average queue length increases but as Erlangian parameter (r) increases, the queue length decreases. This is a quite remarkable observation that analytical and ANFIS results are at par with each other.



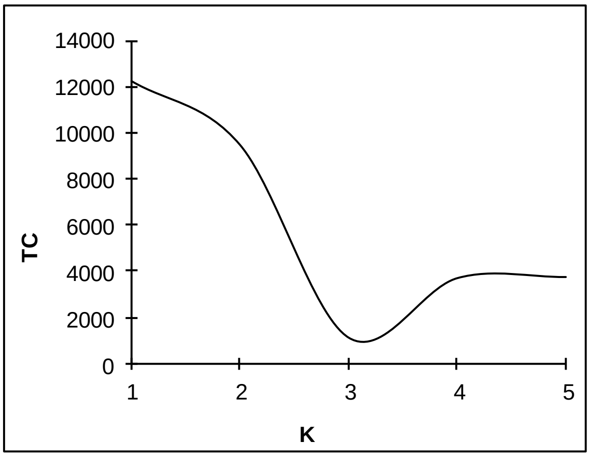
(a)



(b)



(c)



(d)

Fig. 1: Profiles of expected total cost of the system by varying K for set 1, 2, 3 and 4.

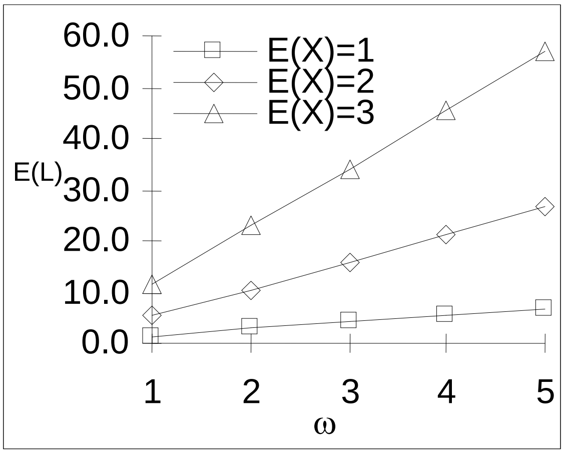


Fig. 2: Graph depicting the effect of ω on E(L) by varying E(X).

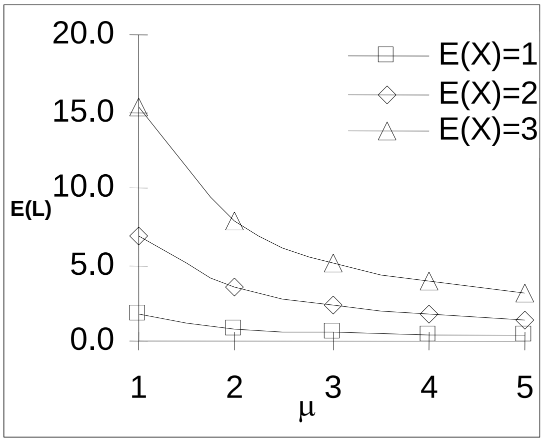


Fig. 2: Graph depicting the effect of ω on E(L) by varying E(X).

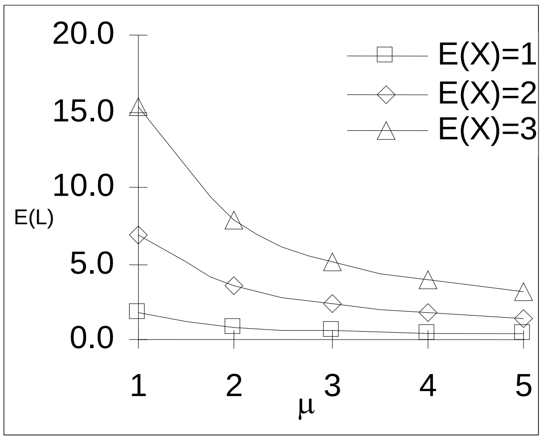


Fig. 3: Graph depicting the effect of μ on E(L) by varying E(X).

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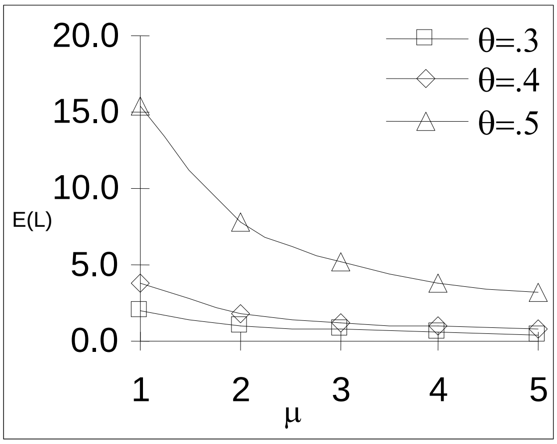

Fig. 4: Graph depicting the effect of μ on E(L) by varying θ.

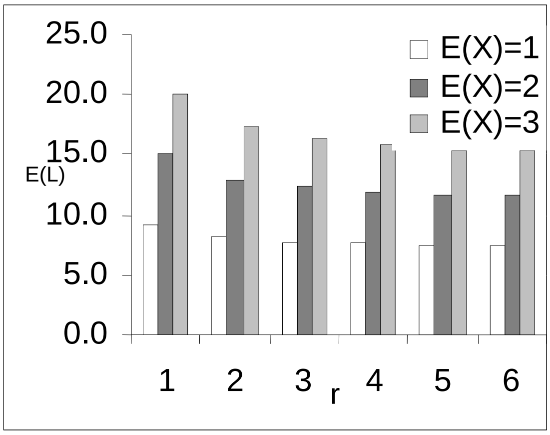

Fig.5: Graph depicting the effect of r on E(L) by varying E(X).

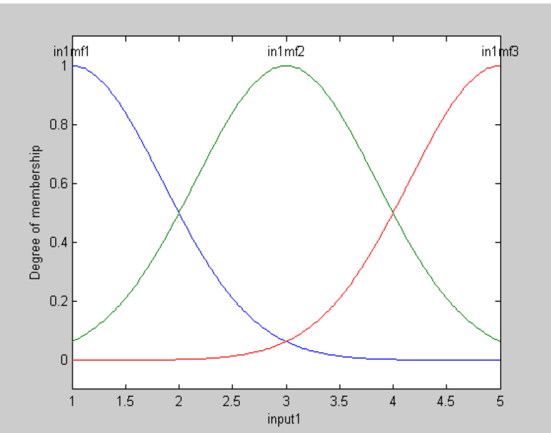

Fig. 6(a): Membership functions for λ

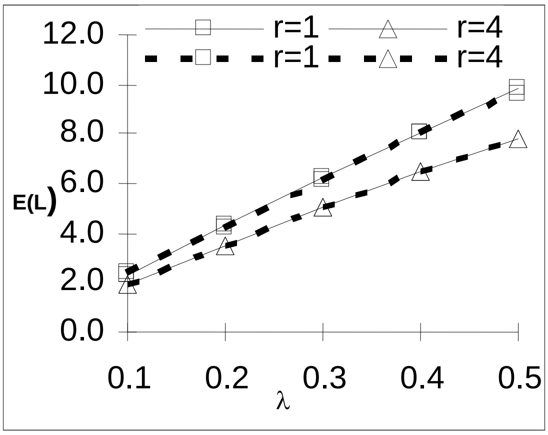

Fig. 6(b): E(L) by varying λ for different values of r.

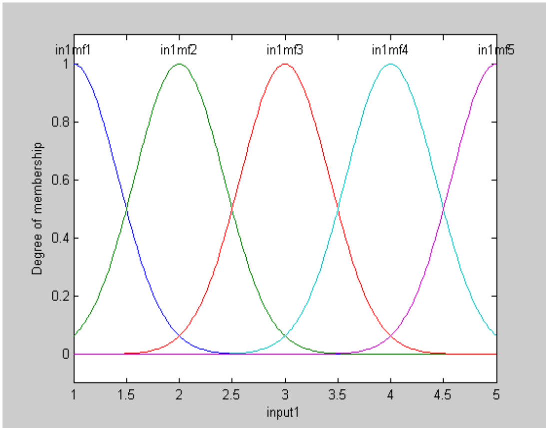

Fig. 7(a): Membership functions for θ .

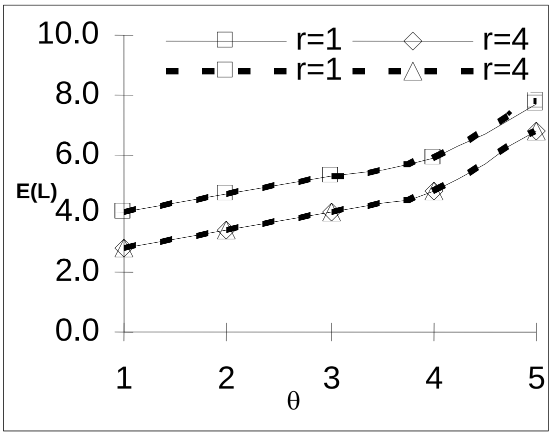

Fig. 7(b): E(L) by varying θ for different values of r.

From the above illustrations, overall we conclude that the expected number of customers in the system increases remarkably as arrival rate and vacation rate increase. The service rate of the server has significant contributions in lowering down the queue length.

Table -1 Cost elements for the different cost sets.

Cost Sets	C_b(Rs)	C_s (Rs)	C_h (Rs)	C_{V0} (Rs)	C_{V1} (Rs)	C_{V2} (Rs)	C_{V3} (Rs)	C_{V4} (Rs)	C_{V5} (Rs)
Set I	1000	1500	500	1000	500	1900	2900	2900	2000
Set II	2000	500	500	1000	500	1900	2900	2900	4000
SetIII	2000	500	500	1000	500	300	500	700	4000
Set IV	3500	200	2000	1500	500	400	700	1000	6000

Table 2: Linguistic values for various input parameters.

Input variable	Number of membership functions	Linguistic Values
Vacation probability (θ)	3	▪ Low ▪ Average ▪ High Large
Arrival rate (λ)	5	▪ Low ▪ Moderate ▪ Average ▪ High ▪ Very high

Table 1 shows the cost element for different cost sets. Table 2 provides the linguistic values of the input parameters and the shapes of the corresponding membership functions for figs 6(b)-7(b); the shape for the same are shown in figs 6(a)-7(a), respectively.

IX. CONCLUDING REMARKS

In this paper an $M^X/G/1$ queue with multi optional vacations and state dependent rates have been studied. For analysis purpose, we apply supplementary variable technique to find the probability generating function of the queueing system, which is further used to establish some performance measures in explicit form. To validate the computational tractability of analytical results, numerical experiment along with sensitivity analysis is performed. We also approximate the performance measures by using a special class of neuro-fuzzy systems, i.e. adaptive Network based Fuzzy Inference System (ANFIS) that can identify parameters by using superposed learning methods. We have incorporated the realistic features of state dependent rates, bulk arrival and optional multi vacation altogether which make our model to deal with more versatile congestion circumstances.

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