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REVISED TRIGONOMETRIC SINE FUNCTION THEOREM

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Abstract— **This research paper is based on new findings and patterns which merge as an alternative to conventional trigonometric theorems. These findings are not based on any pre-theorems in geometry.**

Keywords— **Trigonometry, Mathematical Theorems.**

I. INTRODUCTION

Trigonometry is a sub-branch under the branch of Geometry of Mathematics[1] which aims at establishing relations between sides ratio measures with angles of a Right-angled triangle, however these calculations are way too complicated and long which generally gives inaccurate or undesirable values as answers, this complicated ways can even be harder for invoice learners.As seen whole there are many different complicated ways in trigonometry and one of them is TRIGONOMETRIC EQUATIONS which ought to give values for angles of different trigonometric functions like sine, cosine, tan, etc.

Trigonometric Theorems are complicated and so are their proofs. There were many powerful pre-published researches like BIBHORR FORMULA[2] which helps in finding measure of angle which are opposite to the medium sized side of a Right-Angled Triangle, but there is one limitation, it does not give different angles which form same ratios in the trigonometric functions, without long calculations and so Trigonometric Theorems are used for this heavy work, but this theorems are even complex as discussed above.. However there are new findings which work as an alternative to these theorems and are more efficient, easy to understand and apply.

II. OVERVIEW TO TRIGONOMETRIC FUNCTIONS AND ANGLES

A. Circular Method –

In Fig. 1. The unit circle is divided into 4 different coordinates on the cartesian plane[3], line OA is the initial line and after rotation the terminal line is OB, rotated by few positive radians. Now the perpendicular from point B to line OA would have length equal to the sin function over the radians terminal line OB has been rotated from OA..

Now there are few general but important points to remember which would be used throughout this paper:

- Angles are always measured from positive x-axis
- Angles are measured as whole even if crossing complete angle (360° or 2π)
- Sin function of the angle is measured as equal to the y-coordinate of the point on the circle which terminal line OB touches.
- Cos function of the angle is measured as the x-coordinate form of the point on the circle which terminal line 0B intersects.
- Ratios of Trigonometric functions repeat after complete angle interval.

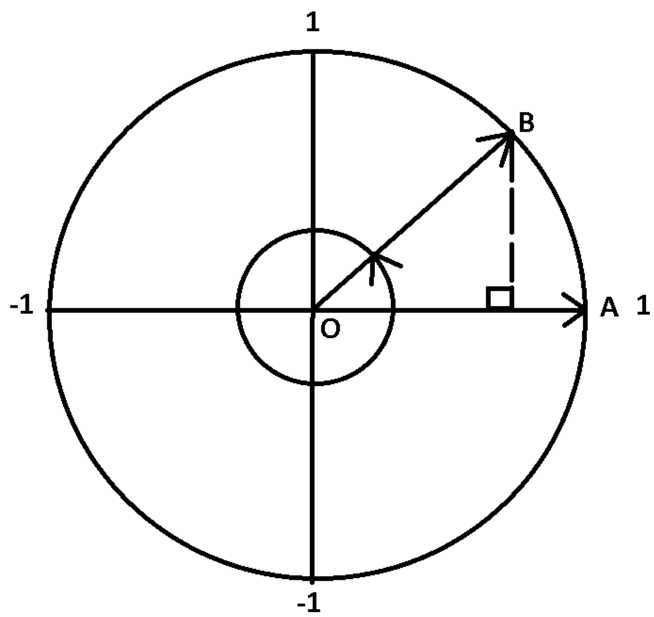

Fig. 1. Unit Circle in coordinate plane.

[1] https://en.wikipedia.org/wiki/Trigonometry
[2] https://innovate.mygov.in/innovation/bibhorr-formula-shortcut-to-trigonometry/
[3] https://en.wikipedia.org/wiki/Unit_circle

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a. **B. Conventional Trigonometric Theorems and their limitations –**

Points to be inferred from Fig. 1.

- For one ratio of trigonometric functions there exists infinite angles. (4th point from general points)
- 4 angles are formed (2 positive and 2 negative) for forming the same ratio of respective functions in single rotation..

As from point 1, one ratio can have multiple angles and for measuring these angles Trigonometric theorems are formed.

1. Conventional Sine Function theorem:

For any x and y if sin(x)=sin(y) then

$$x = n\pi + (-1)^n y$$

As seen above these theorems may look easy to apply but have uncertainty about at what input of n the angle would be in what quadrant and at what sign, plus their proof is also complex and even harder for invoice learners.

Instead of using these theorems, one can use equations which emerge as simple numerical patterns and can be used to find angles forming the same ratio which the function forms when applied to a known angle.

To differentiate from Conventional Trigonometric theorems these new theorems are to be given the prefix of their founder Jyotiraditya Jadhav, an Indian Mathematician.

2. Jadhav's Sine Function Theorem:

For any x and y if sin(x) = sin(y) then

$$y = (2n-1)\pi \mp x$$

Proof : If $\sin(x) > 0$

then $y = \pi - x$

If $\sin(x) < 0$

then $y = 2\pi - (x-\pi)$

giving $y = 3\pi - x$

Now suppose $x > 2\pi$

then y again $= 3\pi - x$

From this we can observe repeating pattern and organize the theorem as $y = (2n-1)\pi \mp x$

Rules to use The Jadhav's Sine Function Theorem

- To get y as positive i.e. other positive angles for which sine ratio remains same, n should always equal to 1 or above. $(2n-1)\pi - x$
- To get y as negative i.e. Negative angle for which sine ratio is additive inverse of , n should always equal to 0 or values below it. $(2n-1)\pi + x$
- This theorem even works if x is a negative angle and would follow the same two rules given above with any sign of angle.

III. PRACTICAL PROOF

Here we will use Jadhav's Sin Function Theorem in practical questions:

1. $\sin(\pi/180) = \sin(y)$

Now we can easily measure all the possible values of y, which is given by $y = (2n-1)\pi \mp x$

Putting n as 1 , $y = \pi-\pi/180$
Putting n as 2 , $y = 3\pi-\pi/180$
Putting n as 3 , $y = 5\pi-\pi/180$
Putting n as 0, $y = -\pi+\pi/180$
Putting n as -1, $y = -2\pi+\pi/180$
Putting n as -2, $y = -5\pi.+\pi/180$
and so on.

2. $\sin(2\pi/180) = \sin(y)$

Now we can easily measure all the possible values of y, which is given by $y = (2n-1)\pi \mp x$

Putting n as 1 , $y = \pi-2\pi/180$
Putting n as 2 , $y = 3\pi-2\pi/180$
Putting n as 3 , $y = 5\pi-2\pi/180$
Putting n as 0, $y = -\pi+2\pi/180$
Putting n as -1, $y = -2\pi+2\pi/180$
Putting n as -2, $y = -5\pi.+2\pi/180$
and so on.

3. $\sin(3\pi/180) = \sin(y)$

Now we can easily measure all the possible values of y, which is given by $y = (2n-1)\pi \mp x$

Putting n as 1 , $y = \pi-3\pi/180$
Putting n as 2 , $y = 3\pi-3\pi/180$
Putting n as 3 , $y = 5\pi-3\pi/180$
Putting n as 0, $y = -\pi+3\pi/180$
Putting n as -1, $y = -3\pi+3\pi/180$
Putting n as -2, $y = -5\pi.+3\pi/180$

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and so on.

4. sin (4π/180) = sin (y)

Now we can easily measure all the possible values of y, which is given by y = (2n-1)π ∓ x

Putting n as 1 , y = π-4π/180
Putting n as 2 , y = 3π-4π/180
Putting n as 3 , y = 5π-4π/180
Putting n as 0, y = -π+4π/180
Putting n as -1, y = -3π+4π/180
Putting n as -2, y = -5π.+4π/180
and so on.

5. sin (5π/180) = sin (y)

Now we can easily measure all the possible values of y, which is given by y = (2n-1)π ∓ x

Putting n as 1 , y = π-5π/180
Putting n as 2 , y = 3π-5π/180
Putting n as 3 , y = 5π-5π/180
Putting n as 0, y = -π+5π/180
Putting n as -1, y = -3π+5π/180
Putting n as -2, y = -5π.+5π/180
and so on.

6. sin (6π/180) = sin (y)

Now we can easily measure all the possible values of y, which is given by y = (2n-1)π ∓ x

Putting n as 1 , y = π-6π/180
Putting n as 2 , y = 3π-6π/180
Putting n as 3 , y = 5π-6π/180
Putting n as 0, y = -π+6π/180
Putting n as -1, y = -3π+6π/180
Putting n as -2, y = -5π.+6π/180
and so on.

7. sin (7π/180) = sin (y)

Now we can easily measure all the possible values of y, which is given by y = (2n-1)π ∓ x

Putting n as 1 , y = π-7π/180
Putting n as 2 , y = 3π-7π/180
Putting n as 3 , y = 5π-7π/180
Putting n as 0, y = -π+7π/180
Putting n as -1, y = -3π+7π/180
Putting n as -2, y = -5π.+7π/180
and so on.

8. sin (8π/180) = sin (y)

Now we can easily measure all the possible values of y, which is given by y = (2n-1)π ∓ x

Putting n as 1 , y = π-8π/180
Putting n as 2 , y = 3π-8π/180
Putting n as 3 , y = 5π-8π/180
Putting n as 0, y = -π+8π/180
Putting n as -1, y = -3π+8π/180
Putting n as -2, y = -5π.+8π/180
and so on.

9. sin (9π/180) = sin (y)

Now we can easily measure all the possible values of y, which is given by y = (2n-1)π ∓ x

Putting n as 1 , y = π-9π/180
Putting n as 2 , y = 3π-9π/180
Putting n as 3 , y = 5π-9π/180
Putting n as 0, y = -π+9π/180
Putting n as -1, y = -3π+9π/180
Putting n as -2, y = -5π.+9π/180
and so on.

10. sin (10π/180) = sin (y)

Now we can easily measure all the possible values of y, which is given by y = (2n-1)π ∓ x

Putting n as 1 , y = π-10π/180
Putting n as 2 , y = 3π-10π/180
Putting n as 3 , y = 5π-10π/180
Putting n as 0, y = -π+10π/180
Putting n as -1, y = -3π+10π/180
Putting n as -2, y = -5π.+10π/180
and so on.

11. sin (11π/180) = sin (y)

Now we can easily measure all the possible values of y, which is given by y = (2n-1)π ∓ x

Putting n as 1 , y = π-11π/180
Putting n as 2 , y = 3π-11π/180
Putting n as 3 , y = 5π-11π/180
Putting n as 0, y = -π+11π/180
Putting n as -1, y = -3π+11π/180
Putting n as -2, y = -5π.+11π/180
and so on.

12. sin (12π/180) = sin (y)

Now we can easily measure all the possible values of y, which is given by y = (2n-1)π ∓ x

Putting n as 1 , y = π-12π/180
Putting n as 2 , y = 3π-12π/180
Putting n as 3 , y = 5π-12π/180
Putting n as 0, y = -π+12π/180
Putting n as -1, y = -3π+12π/180
Putting n as -2, y = -5π.+12π/180
and so on.

13. sin (13π/180) = sin (y)

Now we can easily measure all the possible values of y, which is given by y = (2n-1)π ∓ x

Putting n as 1 , y = π-13π/180
Putting n as 2 , y = 3π-13π/180
Putting n as 3 , y = 5π-13π/180
Putting n as 0, y = -π+13π/180
Putting n as -1, y = -3π+13π/180
Putting n as -2, y = -5π.+13π/180
and so on.

14. sin (14π/180) = sin (y)

Now we can easily measure all the possible values of y, which is given by y = (2n-1)π ∓ x

Putting n as 1 , y = π-14π/180
Putting n as 2 , y = 3π-14π/180
Putting n as 3 , y = 5π-14π/180
Putting n as 0, y = -π+14π/180
Putting n as -1, y = -3π+14π/180
Putting n as -2, y = -5π.+14π/180
and so on.

15. sin (14π/180) = sin (y)

Now we can easily measure all the possible values of y, which is given by y = (2n-1)π ∓ x

Putting n as 1 , y = π-14π/180
Putting n as 2 , y = 3π-14π/180
Putting n as 3 , y = 5π-14π/180
Putting n as 0, y = -π+14π/180
Putting n as -1, y = -3π+14π/180
Putting n as -2, y = -5π.+14π/180
and so on.

16. sin (16π/180) = sin (y)

Now we can easily measure all the possible values of y, which is given by y = (2n-1)π ∓ x

Putting n as 1 , y = π-16π/180
Putting n as 2 , y = 3π-16π/180
Putting n as 3 , y = 5π-16π/180
Putting n as 0, y = -π+16π/180
Putting n as -1, y = -3π+16π/180
Putting n as -2, y = -5π.+16π/180

And like this we can find all the angles for which sine function makes a specific ratio know the angle.

IV. CONCLUSION

We can conclude that Jadhav's Sine Function Theorem can be used as an effective alternative which is easy to understand and has easier proof with better application for finding multiple angles at which the sine function ratio remains same.

V. REFERENCE

[1] https://en.wikipedia.org/wiki/Trigonometry
[2] https://innovate.mygov.in/wp-content/uploads/2018/09/mygov1535992494424559.pdf.
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