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ESTIMATION OF VARIANCE OF TIME TO RECRUITMENT IN A TWO GRADED MANPOWER SYSTEM WITH DIFFERENT EPOCHS FOR DECISIONS AND EXITS AND INTER-DECISION TIMES AS AN ORDER STATISTICS

G. Ishwarya
Assistant Professor, Mathematics,
Agni College of Technology, OMR,
Thalambur, Chennai – 600 130,
Tamil Nadu, India.

A. Srinivasan
Professor Emeritus, PG & Research
Department of Mathematics, Bishop
HeberCollege, Trichy- 620017,
Tamil Nadu, India.

Abstract - In this paper, a two graded manpower system which is subject to exit of personnel due to the policy decisions taken in the system is considered. There is an associated loss of manpower if a person quits. Based on shock model approach, a mathematical model is constructed using a univariate CUM policy of recruitment. The analytical expression for the mean and variance of the time to recruitment are obtained when i) the loss of manpower process for the organization form a sequence of independent and identically distributed exponential random variables ii) the inter-exit times form an ordinary renewal process and iii) the inter-policy decision times as an order statistics. The explicit expressions for the performance measures are derived and relevant conclusions are made.

Keywords: Two grade manpower system; Decision and exit epochs; order statistics; Ordinary renewal process; Univariate CUM policy of recruitment; Mean and variance of time to recruitment.

AMS Mathematics Subject Classification (2010): 60H30

I. INTRODUCTION

In administrative as well as production-oriented organization it is a usual phenomenon that exit of personnel happens whenever policy decision regarding revision of wages, incentives and revised sales and targets are announced. This in turn leads to depletion of manpower, which can be conceptualized in terms of man hours. As the recruitment involves several costs, it is usual that the organization has the natural reluctance to go in for frequent recruitment and hence suitable recruitment planning has to be designed in order to offset the loss in manpower. In the design of the recruitment policies, several authors have used shock model approach in reliability theory. If the total loss or maximum loss of man hours due to the exit of personnel crosses a particular level, known as threshold for the organization, the organization reaches an uneconomic status which otherwise be called the breakdown point and recruitment is to be done at this point. In [1, 2] the authors have discussed several manpower planning models using Markovian and renewal theoretic approach. The problem of finding the time to recruitment for a two graded manpower system using shock model approach was initiated by the authors in [7]. They have studied this problem when the loss of manpower in the organization is maximum (minimum) of thresholds for the loss of manpower in the two grades. Later, several researchers [4], [5], [6], [8], [9] and [10] have studied the problem of time to recruitment for a single and multi grade manpower system under different conditions on the loss of manpower, inter-decision times and the threshold for the loss of manpower using univariate CUM policy of recruitment. The concept of non-instantaneous loss of manpower in decision epochs has been introduced for the first time in [11, 12] for a single grade manpower system and the performance measures are obtained for the same when the inter-decision times are independent and identically distributed exponential random variables using different probabilistic analysis. In [13], the authors have extended the research work in [12] when the inter decision times form an order statistics by using indicatory function technique. For a two graded manpower system, the authors in [14, 15, 16 , 17 & 18], have extended the work of [10] to obtain the performance measures according as the inter decision times are independent and identically distributed exponential random variables or exchangeable and constantly correlated exponential random variables or forming a geometric process using the above cited techniques. The present research work is the extension of the work in [18] when the inter-decision times form an order statistics.

II. MODEL DESCRIPTION AND ANALYSIS

Consider an organization with two grades (grade-1and grade-2) taking policy decisions at random epochs in (0, ∞) and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower, if a person quits. It is assumed that the epochs for decisions and exits are different and the loss of manpower is linear and cumulative. For i=1,2,3…, let X_i be independent and identically distributed exponential random variables representing the amount of depletion of manpower (loss of man hours) in the organization at the i[th] exit point with probability distribution $M(.)$, density function $m(.)$ and mean $\frac{1}{\lambda}$ (λ >0). Let S_i be the cumulative loss of manpower up to i-th exit. Let U_i be the time between (i-1)th and i[th] policy decisions, forming a sequence of independent and identically distributed random variables with probability distribution function F(.) , density function f(.). Let $F_{U_{(j)}}(.)$ and $f_{U_{(j)}}(.)$ be the distribution and the probability density function of the j[th] order statistics (j=1,2,…,n) selected from the sample of size 'n' from the population $\{U_i\}_{i=1}^{\infty}$. Let R_i be the time between (i-1)[th] and i[th] exits. It is assumed that R_i's are independent and identically distributed random variables with distribution function $G(.)$ and density function $g(.)$. Let D_{i+1} be the waiting time up to (i+1)[th] exit. Let $E(R)$ and $V(R)$ be the mean and variance of the inter-exit times respectively. Let Y_1, Y_2 be continuous random variables representing the thresholds for the cumulative loss of man hours in grades 1 and 2 respectively. Let Y be the breakdown threshold for the cumulative loss of manhours in the organization with distribution function $H(.)$ and density function $h(.)$. Let q (q≠0) be the probability that every policy decision produces an attrition. Let $I(A)$ be the indicatory function of the event A. Let T be a continuous random variable denoting the time for recruitment with mean $E(T)$ and variance $V(T)$.

The univariate CUM policy of recruitment employed in this paper is stated as follows: ***Recruitment is done whenever the cumulative loss of man hours in the organization exceeds the breakdown threshold Y***.

We now obtain the variance of time to recruitment. By the probabilistic arguments, the time to recruitment can be written as

$$T = \sum_{i=0}^{\infty} D_{i+1}\, I(S_i \le Y < S_{i+1})$$

and

$$E(T) = E(R)\sum_{i=0}^{\infty}(i+1)\, P(S_i \le Y < S_{i+1}) \quad \textbf{………(1)}$$

Similarly $T^2 = \sum_{i=0}^{\infty} {D_{i+1}}^2\, I(S_i \le Y < S_{i+1})$

and

$$E(T^2) = V(R)\sum_{i=0}^{\infty}(i+1)\, P(S_i \le Y < S_{i+1}) + [E(R)]^2\sum_{i=0}^{\infty}(i+1)^2\, P(S_i \le Y < S_{i+1}) \quad \textbf{…...(2)}$$

By the law of total probability,

$$P(S_i \le Y < S_{i+1}) = P(0 \le Y - S_i < S_{i+1} - S_i)$$
$$= \int_0^{\infty}\int_0^{\infty} P(X_{i+1} > y - x)\, dM_i(x)\, dH(y)$$
$$= \int_0^{\infty}\left[\int_0^{y} P(X_{i+1} > y - x)\, dM_i(x)\right] dH(y)$$
$$P(S_i \le Y < S_{i+1}) = \int_0^{\infty}\left[\int_0^{y} \overline{M}(y - x)\, dM_i(x)\right] dH(y) \quad \textbf{……...(3)}$$

It can be proved that

$$G(x) = \sum_{n=1}^{\infty} q(1-q)^{n-1} F_n(x) \quad \textbf{…..……(4)}$$

From [3], the density function of $U_{(j)}$ is given by

$$f_{U_{(j)}}(x) = \frac{n!}{(n-j)!(j-1)!}\left[F(x)\right]^{j-1}\left[1 - F(x)\right]^{n-j} f(x) \quad \textbf{…………(5)}$$

We now obtain the explicit expression for E(T) and E(T^2)

Case (i): (j=1) $f(t) = f_{U_{(1)}}(t)$

Here $f(t) = n\theta\left(e^{-\theta x}\right)^n$ (by (5))

For this case, From (4), it can be proved that

$$E(R)=\frac{1}{n\theta q},\ E(R^2)=\frac{2}{n^2\theta^2q^2} \quad \text{and}$$

$$V(R)=\frac{1}{n^2\theta^2q^2} \qquad \text{.......(6)}$$

We now consider different forms for Y by assuming that Y_1 and Y_2 follow exponential distribution with parameters α_1,α_2 respectively.

Suppose $Y=Max(Y_1,Y_2)$

In this case

$$H(y)=1-e^{-\alpha_1 y}-e^{-\alpha_2 y}+e^{-(\alpha_1+\alpha_2)y}$$

$$P(S_i\le Y<S_{i+1})=\frac{\alpha_1}{\lambda+\alpha_1}\left(\frac{\lambda}{\lambda+\alpha_1}\right)^i+\frac{\alpha_2}{\lambda+\alpha_2}\left(\frac{\lambda}{\lambda+\alpha_2}\right)^i-\frac{\alpha_1+\alpha_2}{\lambda+\alpha_1+\alpha_2}\left(\frac{\lambda}{\lambda+\alpha_1+\alpha_2}\right)^i \qquad \text{..........(7)}$$

Using (6) and (7) in (1) & (2) and after simplification, we get

$$E(T)=\left(\frac{1}{n\theta q}\right)\left[\frac{\lambda+\alpha_1}{\alpha_1}+\frac{\lambda+\alpha_2}{\alpha_2}-\frac{\lambda+\alpha_1+\alpha_2}{\alpha_1+\alpha_2}\right] \qquad \text{.........(8)}$$

$$E(T^2)=\left(\frac{1}{n\theta q}\right)^2\left[\frac{\lambda+\alpha_1}{\alpha_1}+\frac{\lambda+\alpha_2}{\alpha_2}-\frac{\lambda+\alpha_1+\alpha_2}{\alpha_1+\alpha_2}\right]+\left(\frac{1}{n^2\theta^2q^2}\right)\left[\begin{array}{l}\frac{(\lambda+\alpha_1)(2\lambda+\alpha_1)}{\alpha_1^2}+\frac{(\lambda+\alpha_2)(2\lambda+\alpha_2)}{\alpha_2^2}\\ -\frac{(\lambda+\alpha_1+\alpha_2)(2\lambda+\alpha_1+\alpha_2)}{(\alpha_1+\alpha_2)^2}\end{array}\right] \qquad \text{........(9)}$$

$$V(T)=E(T^2)-\left[E(T)\right]^2 \qquad \text{........(10)}$$

Equation (10) together with equations (8) and (9) will give V(T) for this case.

Suppose $Y=Min(Y_1,Y_2)$

In this case, $H(y)=1-e^{-(\alpha_1+\alpha_2)y}$

$$P(S_i\le Y<S_{i+1})=\frac{\alpha_1+\alpha_2}{\lambda+\alpha_1+\alpha_2}\left(\frac{\lambda}{\lambda+\alpha_1+\alpha_2}\right)^i \qquad \text{........(11)}$$

Using (6) and (11) in (1) & (2), we get

$$E(T)=\left(\frac{1}{n\theta q}\right)\left[\frac{\lambda+\alpha_1+\alpha_2}{\alpha_1+\alpha_2}\right] \qquad \text{.........(12)}$$

$$E(T^2)=\left(\frac{1}{n\theta q}\right)^2\left[\frac{\lambda+\alpha_1+\alpha_2}{\alpha_1+\alpha_2}\right]+\left(\frac{1}{n^2\theta^2q^2}\right)\left[\frac{(\lambda+\alpha_1+\alpha_2)(2\lambda+\alpha_1+\alpha_2)}{(\alpha_1+\alpha_2)^2}\right] \qquad \text{.........(13)}$$

Equations (12) and (13) together with (10) will give V(T) for this case.

Suppose $Y=Y_1+Y_2$

In this case

$$H(y)=1+\frac{\alpha_2}{\alpha_1-\alpha_2}e^{-\alpha_1 y}-\frac{\alpha_1}{\alpha_1-\alpha_2}e^{-\alpha_2 y}$$

$$P(S_i\le Y<S_{i+1})=\frac{\alpha_1}{\alpha_1-\alpha_2}\left(\frac{\alpha_2}{\lambda+\alpha_2}\right)\left(\frac{\lambda}{\lambda+\alpha_2}\right)^i-\frac{\alpha_2}{\alpha_1-\alpha_2}\left(\frac{\alpha_1}{\lambda+\alpha_1}\right)\left(\frac{\lambda}{\lambda+\alpha_1}\right)^i \qquad \text{.........(14)}$$

$$E(T)=\left(\frac{1}{n\theta q}\right)\left[\frac{\alpha_1}{(\alpha_1-\alpha_2)}\left(\frac{\lambda+\alpha_2}{\alpha_2}\right)-\frac{\alpha_2}{(\alpha_1-\alpha_2)}\left(\frac{\lambda+\alpha_1}{\alpha_1}\right)\right] \qquad \text{..........(15)}$$

$$E(T^2)=\left(\frac{1}{n\theta q}\right)^2\left[\frac{\alpha_1}{(\alpha_1-\alpha_2)}\left(\frac{\lambda+\alpha_2}{\alpha_2}\right)-\frac{\alpha_2}{(\alpha_1-\alpha_2)}\left(\frac{\lambda+\alpha_1}{\alpha_1}\right)\right]+\left(\frac{1}{n^2\theta^2q^2}\right)\left[\frac{\alpha_1(\lambda+\alpha_2)(2\lambda+\alpha_2)}{(\alpha_1-\alpha_2)\alpha_2^2}-\frac{\alpha_2(\lambda+\alpha_1)(2\lambda+\alpha_1)}{(\alpha_1-\alpha_2)\alpha_1^2}\right] \qquad \text{.........(16)}$$

Equation (10) together with equations (15) and (16) will give V(T) for this case.

Case (ii): (j=n) $f(t)=f_{U_{(n)}}(t)$

From (5),

$$f(t)=n[F(x)]^{n-1}f(x)=n\theta e^{-\theta x}\left(1-e^{-\theta x}\right)^{n-1}$$

Again from (4), we can prove that $E(R)=\frac{1}{\theta q}\left(\sum_{j=1}^{n}\frac{1}{j}\right)$,

$$E(R^2)=\frac{1}{\theta^2q}\left(\sum_{j=1}^{n}\frac{1}{j^2}\right)+\left(\frac{2-q}{\theta^2q^2}\right)\left(\sum_{j=1}^{n}\frac{1}{j}\right)^2 \quad \text{and}$$

$$V(R)=\frac{1}{\theta^2q^2}\left(\sum_{j=1}^{n}\frac{1}{j^2}\right)+\left(\frac{1-q}{\theta^2q^2}\right)\left(\sum_{j=1}^{n}\frac{1}{j}\right)^2$$

For this case also we can find explicit expression for E(T) and $E(T^2)$ by assuming different forms for Y.

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For the model $Y = Max(Y_1, Y_2)$, using (1) and (2),

$$E(T) = \left(\frac{1}{\theta q}\left(\sum_{j=1}^{n}\frac{1}{j}\right)\right)\left[\frac{\lambda+\alpha_1}{\alpha_1}+\frac{\lambda+\alpha_2}{\alpha_2}-\frac{\lambda+\alpha_1+\alpha_2}{\alpha_1+\alpha_2}\right]$$

………(17)

$$E(T^2) = \left(\frac{1}{\theta^2 q^2}\left(\sum_{j=1}^{n}\frac{1}{j^2}\right)+\left(\frac{1-q}{\theta^2 q^2}\right)\left(\sum_{j=1}^{n}\frac{1}{j}\right)^2\right)\left[\frac{\lambda+\alpha_1}{\alpha_1}+\frac{\lambda+\alpha_2}{\alpha_2}-\frac{\lambda+\alpha_1+\alpha_2}{\alpha_1+\alpha_2}\right] + \left(\frac{1}{\theta^2 q^2}\left(\sum_{j=1}^{n}\frac{1}{j}\right)^2\right)\left[\frac{(\lambda+\alpha_1)(2\lambda+\alpha_1)}{\alpha_1^2}+\frac{(\lambda+\alpha_2)(2\lambda+\alpha_2)}{\alpha_2^2}-\frac{(\lambda+\alpha_1+\alpha_2)(2\lambda+\alpha_1+\alpha_2)}{(\alpha_1+\alpha_2)^2}\right]$$

………..(18)

For the model $Y = Min(Y_1, Y_2)$

$$E(T) = \left(\frac{1}{\theta q}\left(\sum_{j=1}^{n}\frac{1}{j}\right)\right)\left[\frac{\lambda+\alpha_1+\alpha_2}{\alpha_1+\alpha_2}\right]$$

…………(19)

$$E(T^2) = \left(\frac{1}{\theta^2 q^2}\left(\sum_{j=1}^{n}\frac{1}{j^2}\right)+\left(\frac{1-q}{\theta^2 q^2}\right)\left(\sum_{j=1}^{n}\frac{1}{j}\right)^2\right)\left[\frac{\lambda+\alpha_1+\alpha_2}{\alpha_1+\alpha_2}\right] + \left(\frac{1}{\theta^2 q^2}\left(\sum_{j=1}^{n}\frac{1}{j}\right)^2\right)\left[\frac{(\lambda+\alpha_1+\alpha_2)(2\lambda+\alpha_1+\alpha_2)}{(\alpha_1+\alpha_2)^2}\right]$$

…………(20)

For the model $Y = Y_1 + Y_2$

$$E(T) = \frac{1}{\theta q}\left(\sum_{j=1}^{n}\frac{1}{j}\right)\left[\frac{\alpha_1}{(\alpha_1-\alpha_2)}\left(\frac{\lambda+\alpha_2}{\alpha_2}\right)-\frac{\alpha_2}{(\alpha_1-\alpha_2)}\left(\frac{\lambda+\alpha_1}{\alpha_1}\right)\right]$$

……..(20)

$$E(T^2) = \left(\frac{1}{\theta^2 q^2}\left(\sum_{j=1}^{n}\frac{1}{j^2}\right)+\left(\frac{1-q}{\theta^2 q^2}\right)\left(\sum_{j=1}^{n}\frac{1}{j}\right)^2\right)\left[\frac{\alpha_1}{(\alpha_1-\alpha_2)}\left(\frac{\lambda+\alpha_2}{\alpha_2}\right)-\frac{\alpha_2}{(\alpha_1-\alpha_2)}\left(\frac{\lambda+\alpha_1}{\alpha_1}\right)\right] + \frac{1}{\theta^2 q^2}\left(\sum_{j=1}^{n}\frac{1}{j}\right)^2\left[\frac{\alpha_1(\lambda+\alpha_2)(2\lambda+\alpha_2)}{(\alpha_1-\alpha_2)\alpha_2^2}-\frac{\alpha_2(\lambda+\alpha_1)(2\lambda+\alpha_1)}{(\alpha_1-\alpha_2)\alpha_1^2}\right]$$

………..(21)

From equation (10), we can obtain explicit expressions of V(T) for different models of case (ii).

III. CONCLUSION

The model discussed in this paper are found to be more realistic and new for a two grade manpower system in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs (ii) associating a probability for any decision to have exit points and (iii) inter-decision times form an order statistics. From the organization's point of view, our models are more suitable than the corresponding models with instantaneous attrition at decision epochs, as the provision of exit points at which attrition actually takes place, postpone the time to recruitment.

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