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THE ACCURACY OF ZERNIKE RECONSTRUCTION OF OCULAR ABERRATIONS WITH MEASUREMENT NOISE

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Abstract- **The ocular aberrations (wavefront) can be used to describe the vision defects, and wavefront sensing devices (aberrometers) are used to measure the ocular aberrations (OAs). The Zernike polynomials (ZPs) have been successfully used in wavefront reconstruction algorithms of aberrometers for years. As the aberrometer measurements consist with noise, the measurements observed using aberrometers are not always accurate. It is therefore necessary to take multiple measurements from the patient in each sitting. However, taking multiple measurements also causes to induce the variations on measurements, and this variability of measurements is significant, and leads errors in clinical practice. Therefore, the variability should be taken into account during wavefront reconstruction. Consequently, unlike prior work, the accuracy of Zernike representation for the ocular aberrations with noise (due to the system noise and the variability of measurements) is studied. In the study, the noisy data is fitted using ZPs. Data sets are created using extracted data from arbitrary OAs, that is, synthetic data sets are used. Normally distributed very small random numbers are added into the data sets in order to create measurement noise. The magnitude of added noisy has been changed to range the signal-to-noise ratio (SNR) from 55 dB to 20 dB. The corresponding Zernike coefficients for each noised wavefront are computed, and visual acuity is used to examine the deviation of the reconstructed wavefront from the data set. The study concludes that the Zernike polynomials are not good enough to represent the wavefront with noisy level less than certain value of SNR which is greater than the reference SNR values of aberrometers, and note that the aberrometer SNR is always between 20-30 dB.**

Keywords- **Zernike polynomials, Ocular aberrations, Measurement noise, Visual acuity**

I. INTRODUCTION

Optical aberrations can be raised due to optical imperfections of optical systems [9]. Similarly, such imperfections of human eye create ocular aberrations (OAs), which can be used to explain optical irregularity of the eye. Aberrometers are used to measure these OAs [3]. However, the information derived by such optical instruments (aberrometers) should not be considered as precise due to measurement noise. In addition, wavefront data used in clinical care should not be extracted from a single measurement, since it represents only a static snapshot of a dynamically changing aberration pattern. The repeated measurements should be therefore taken for the patient in each sitting. However, the repeated measurements may induce the variation on measurements [4, 5], and this variance can cause errors in custom corrections (corneal ablation, contact lens design).

The Zernike polynomials (ZPs) have widely been used to report the OAs of the eye, and explained in Section 2.2 and Section 2.3.

Accordingly, unlike prior studies, we study that the accuracy of the Zernike representation for the wavefront with noise. Therefore, we use ZPs to construct a smooth surface $y = f(x)$ which fits the data points $(x_i, y_i + \varepsilon_i)$, $i = 1,2,3, \dots, n$ where $x_i = (r_i, \theta_i) \in B_1(0)$ and $y_i \in \mathbb{R}$. The data is noisy with $\{\epsilon_i\}$ independent and identically distributed Gaussian variables with mean 0 and the standard deviation σ.

II. STATE OF THE ART

2.1 Variability of Measurements

Ginis et al. (2004) have conducted a research on variability of wavefront aberration measurements in small pupil sizes using a Shack-Hartmann aberrometer (SHA) [13, 14]. They found that wavefront aberration exhibits a considerable variance near the edge of the pupil while the central part appears more consistently and considerable variability in low and higher order aberrations at different pupil sizes. In order to assess the repeatability of measurements of ocular aberrations and the potential effect of measurement error on custom corneal correction due to this variability, Davies et al. (2003) carried out a study, and the study observed that there were highly statistically significant differences in a large number of Zernike modes between the sets of measurements. In addition, the variability of OA measurements has been studied [6, 12]. Consequently, the variation of OA measurements is considerable and could not be ignored, that is, such variation should be accounted in the wavefront reconstruction process. It is led to improve the vision corrections (laser ablations and contact lens designs).

2.2 Representation of Ocular Aberrations

The OAs can be used to monitor and optimize vision defects. In particular, most of refractive surgeries are rely on the OAs. Accuracy of the refractive surgery is therefore determined by the representation of OAs. Consequently, the representation of OAs plays a crucial role in the field of vision correction [1]. There are different mathematical representations to describe OAs [16]. However, due to the orthogonality and aberration balancing properties, Zernike representation is widely used to express OAs [2, 7, 8, 9, 10]. Also, the ZPs have become a standard for describing OAs of the human eye [15].

2.3 Zernike Representation

In Zernike representation, the ocular aberrations $W(r,\theta)$ with pupil radius R expand in terms of Zernike polynomials $Z_i(\rho,\theta)$ as:

$$W(r,\theta)=\sum_{i=0}^{\infty} c_i Z_i(\rho,\theta), \qquad (1)$$

where $\rho = r/R$ is the normalized pupil radius, and c_i is the i^{th} Zernike coefficient. To be specific, let $W(r_i,\theta_i)$, $i = 1,2,\dots,n$ be n number of elevation data points extracted from an ocular aberration. The given data set can then be fitted by a finite series of Zernike polynomials [11]:

$$W(r_i,\theta_i)=\sum_{i=0}^{J} c_i Z_i(\rho_i,\theta_i)+\epsilon_i, \qquad (2)$$

where ϵ_i is the measurement and modeling error (noise). The corresponding linear model is given:

$$\underline{\boldsymbol{W}} = Z\underline{\boldsymbol{c}} + \underline{\epsilon}. \qquad (3)$$

The coefficient vector $\boldsymbol{c}$ can be estimated using the least squares approach, that is,

$$\hat{\underline{\boldsymbol{c}}} = (Z^T Z)^{-1} Z^T \underline{\boldsymbol{W}}. \qquad (4)$$

III. METHODOLOGY

3.1 Data Collection

The raw data are not available for our study because of some proprietary reasons. Therefore, we have conducted a thought experiment. Also, height data of OAs are used in the study. Data sets are created using extracted data from arbitrary OAs, that is, synthetic data sets are used. Normally distributed very small random numbers are added into data sets in order to create measurement noise which represents the measurement noise and variability.

3.2 Calculating Zernike Coefficients

The elevation data points extracted from an OA can then be fitted using Eq. (2). The corresponding linear model is given by Eq. (3). The coefficient vector $\boldsymbol{c}$ in Eq. (3) can be estimated using the Eq. (4).

Visual acuity is used as a tool to discuss the accuracy of the Zernike reconstruction of OAs with measurement noise. To do so, corresponding visual acuity measurement (VAM) models are obtained and which can be generated by convolving the point spread functions (PSFs) with a target image. The Section 3.3 describes the computation of VAM models.

3.3 Point Spread Function and Visual Acuity Model

PSF shows the degradation of a point source due to the OAs. The OA can therefore be utilized to compute the PSF. Let $W(x,y)$ be the OA. The complex-valued generalized pupil function is then given [17]:

$$P(x,y) = P'(x,y)\exp\left[\frac{2\pi i}{\nu} W(x,y)\right], \qquad (5)$$

where $P'(x,y)$ is the pupil function which is 1 within the pupil and 0 elsewhere, and ν denotes the wavelength. The squared modulus of the Fourier transformation of the generalized pupil function gives the PSF for the given OA:

$$\mathrm{PSF}(x,y) = \|F[P(x,y)]\|^2, \qquad (6)$$

where F denotes the Fourier transformation. Once PSF is calculated, one can compute the retinal image $I(x,y)$ as the convolution of the source image $S(x,y)$ and the PSF:

$$I(x,y) = \mathrm{PSF}(x,y) \otimes S(x,y). \qquad (7)$$

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In this paper, the retinal image quality is used to discuss the variation of visual acuity.

IV. RESULTS & DISCUSSION

A few results are only here presented. However, the results for other synthetic data sets are followed the similar behavior. We have added normally distributed random numbers, (mean=0, $\sigma^2 = 1.09\ \mu$m) into the synthetic data sets to create the noise which represents all sources of noise. The magnitude of added noise has been changed to range the SNR from 55.00 dB to 20.00 dB. Using Eq. (3) and Eq. (4), the corresponding Zernike coefficients for each noised wavefront was computed, and the corresponding VAM models are also obtained. It was used to represent the deviation of the reconstructed wavefront from the data set. Figure 1 shows the change of visual acuity due to the deviation of reconstruction.

SNR = 50.6956 dB SNR = 44.6753 dB SNR = 41.1535 dB dB SNR = 38.6547 dB SNR = 36.7165

Figure 1: The visual acuities corresponding to deviations of the reconstructed wavefronts are shown.

SNR = 52.7288 dB SNR = 46.7082 dB SNR = 40.6876 dB dB SNR = 37.1658 dB SNR = 34.6670

Figure 2: The visual acuities corresponding to deviations of the reconstructed wavefronts are shown

Note that from Figure 1, the visual acuity is decreasing when the SNR is decreasing. However, after a certain value of SNR (38.6547 dB), the visual acuity remains unchanged, that is, the ZPs are not good enough to capture the noise level less than 38.6547 dB. Also, we observed that similar behavior for other data sets. For instance, Figure 2 illustrates the visual acuity distribution with deviation of reconstruction.

V. CONCLUSION

In clinical practice, multiple measurements should be taken from a patient in each sitting using an aberrometer. However, multiple measurements cause to induce the variation on measurements. This variance is significant and should be taken into account to find the single reading for a patient. Also, aberrometer misalignment leads to create noise on the measurement. In each case, currently the ZPs are being used to reconstruct the wavefront for vision corrections. However, we realized that the Zernike polynomials are not good enough to capture the noise induced due to the variability of measurements and aberrometer misalignment.

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