



# DEVELOPMENT OF LINEAR MODEL FOR LIQUID HAMMER FORMING USING DESIGN OF EXPERIMENTS

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**Abstract**— Liquid hammer is a pressure surge or wave caused when a fluid in motion is forced to stop or change direction suddenly which also involves a momentum change. Liquid hammer forming is a forming process in which a dead weight is dropped freely from a certain height over a plunger working inside a cylinder containing liquid generates shock waves which reach the sheet metal kept at a lower part of the cylinder and deforms it to the shape of the die placed below the sheet metal. In the current work, the concept of liquid hammer forming has been demonstrated using water column as the energy transmitting medium to carry out the deformation of an aluminum sheet metal. The die fabricated is of multi-cavity type and a uniform deformation has been seen. The process variables that were considered which affect deformation are the thickness of the specimen, height of drop hammer, number of blows and weight of the drop hammer. In the present work, two factorial design of experiments technique is used in order to find the effect of input parameters on deformation. In design of experiments, number of trails to be conducted is determined by factorial method and design matrix is constructed. After getting the design matrix, regression coefficients are calculated. Adequacy of model is tested by fisher test at 5% significance level. Student's t-test is carried out to check the significance of each regression coefficient. Hence the final equation of deformation is obtained.

**Keywords**— DOE, ANOVA, Factors, Liquid Hammer Forming

## I. INTRODUCTION

### LIQUID HAMMER FORMING:

Liquid hammer process of forming is a simple high energy rate forming technique. In Liquid (water) hammer method of forming sheet metals; the necessary high energy is obtained from the shock waves generated in a liquid column by the impact of a falling weight transmitted through a plunger. The plunger works inside a cylinder containing water of hydraulic fluid generates shock waves which reach the sheet metal kept at the lower part of cylinder and deforms it to the shape of the die placed below the sheet metal. Water can be considered as

homogeneous fluid incapable of supporting any shear; this implies that if the water is loaded impulsively its volume is able to readjust by means of flow through any boundary displacements caused by the loading. Change in the pressure results in change in volume. In this way pressure suddenly applied locally will be transmitted to other points in the fluid as an “elastic wave of disturbance” this will travel at the velocity of sound in the liquid. Since real fluids sustain no shear it is equal to

$$\sqrt{(k / \rho)}$$

Where  $k$  = bulk modulus of the fluid,  $\rho$  = density of the fluid

## II. FULL FACTORIAL DESIGN OF EXPERIMENTS

The full factorial design of an experiment is the procedure of selecting the number of trials and conditions for running them, essential and sufficient for solving the problems that has been set with the required precision. Factorial designs are widely used in experiments involving several factors where it is necessary to investigate the joint effect of the factors on a response.

### 2.1 Objective & Methodology

The mathematical model is developed by using factorial design of experiments to predict the deformation that is caused due to water hammer forming technique. The four factors, namely the height of the drop hammer, thickness of metal sheet, number of blows of the hammer, and weight are analyzed simultaneously by main effects with two or three factors interactions. The developed model is tested for its adequacy and significance of each coefficient is checked by student's t-test at 5% significance level.

The investigation is planned with the following objectives

1. Postulation of mathematical model for deformation



2. Adoption of two level factorial design of experiments and selection of test regions for the variables (factors).
3. Conducting the experiments as per design matrix.
4. Estimation of coefficients of postulated model.
5. Analysis of results
6. Checking the adequacy of the postulated model by 'F-test'
7. Testing the significance of each coefficient of the model by 't-test'
8. Determination of percentage contribution of each factor.

**2.2 Postulation of Model for Deformation**

The factors that are identified to develop a mathematical model in order to predict the deformation of the given specimen are *height of the drop hammer, thickness of the specimen, number of blows* given to the specimen by drop hammer and *weight*. The first order model is assumed with two and three factor interaction which can be expressed as

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{34}X_3X_4 + b_{123}X_1X_2X_3 + b_{124}X_1X_2X_4 + b_{134}X_1X_3X_4 + b_{234}X_2X_3X_4 + b_{1234}X_1X_2X_3X_4$$

Where 'Y' represents deformation of the specimen, X<sub>1</sub>, X<sub>2</sub>, X<sub>3</sub>, X<sub>4</sub> represents thickness of the specimen, height of the drop hammer, number of blows, and weight respectively and b<sub>0</sub>, b<sub>1</sub>... b<sub>1234</sub> are regression coefficients. A two level full factorial design of experiments is adopted for calculating the main and the interaction effects of the 4 factors at 2 levels; 2<sup>4</sup> = 16 experiments are conducted to fit an equation. The design plan with high and low limits as indicated Table -1

**Table-1** Factors and levels

Factors	Designation form		Test levels		Avg.	Variation Interval
	Natural	Coded	Low	High		
Thickness of specimen (mm)	t	X <sub>1</sub>	1.024	1.62	1.32	0.302
Height of the hammer (cm)	h	X <sub>2</sub>	125	175	150	25
Number of blows	b	X <sub>3</sub>	1	2	1.5	0.5
Weight (kg)	w	X <sub>4</sub>	5	7	6	1

**III. MODEL DEVELOPMENT**

In the present work, sequences of steps followed in development of model are

**3.1 Calculation of regression coefficients:**

Here the number of replications for the response are two i.e., Y<sub>1</sub> and Y<sub>2</sub> and average of these is 'Y'. Regression coefficients b<sub>0</sub>, b<sub>1</sub>, b<sub>2</sub>, b<sub>12</sub> etc., are calculated by using the formula given below

$$b_j = [\sum x_{ij} y_i] / N$$

Where N = number of trails (N=16)

Fisher test for adequacy of model (f-test for 5% significance level)

Variance for reproducibility

$$S_y^2 = [2\sum (delY)^2] / N$$

N=number of trails, delY = (Y<sub>1</sub>-Y)

Variance of adequacy,

$$S_{ad}^2 = [2\sum (Y - Y_p)^2] / DOF$$

Y<sub>p</sub> = predicted response.

$$Y_p = b_0x_0[i] + b_1x_1[i] + b_2x_2[i] + \dots$$

Where DOF = degree of freedom

$$= [N - (k + 1)]$$

Where N = number of trails

k = number of factors

$$F\text{-model} = S_{ad}^2 / S_y^2$$

For given values of f<sub>1</sub> and f<sub>2</sub>, F-table value is found from fisher table.

Here f<sub>1</sub>=N-(k+1), f<sub>2</sub>=N

If F-model ≤ F-table, model is adequate in linear form otherwise it is not adequate.

**3.2 Student's t-test (for 5% significance level):**

When the model is adequate in linear form, then t-test is to be conducted to test the significance of each Regression coefficient.

Standard deviation of each coefficient,

$$S_{bj} = \sqrt{(S_y^2 / N)}$$

$$T\text{-ratio} = |b_j| / (S_{bj})$$

For f = N, t value is to be taken from t-table and compared with t-ratio of each regression coefficient. If t-ratio ≥ t-table corresponding regression coefficient is significant. Non-significant coefficients are to be eliminated from the model to arrive the final form of mathematical model in linear form as

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{34}X_3X_4 + b_{123}X_1X_2X_3 + b_{124}X_1X_2X_4 + b_{134}X_1X_3X_4 + b_{234}X_2X_3X_4 + b_{1234}X_1X_2X_3X_4$$

#### IV. EXPERIMENTATION

The equipment is arranged on rectangular concrete base wooden block is placed over the concrete base so as to absorb the shock load without damaging the system die holder is kept on the wooden block. Suitable die is selected and placed in the die holder over which the sheet metal (work piece) is located in the shallow counter sunk bore, of the same diameter as the specimen, on the die face. Venting is provided both in the die and die holder, to prevent spring back of the specimen due to compressing of trapped air. Now the cylinder with two 'O' rings are fixed in order to prevent leakage of the working fluid. Cylinder flange and guide bush are mounted on the cylinders in such a way that it is fixed with a m-10 Allen screw. Two swing bolts are used to clamp the cylinder flange and hold the cylinder in position. Plunger along with 'O' ring fixed on it, is inserted through the guide bush into the cylinder. The mating faces of plunger and cylinder were grinded for smooth movement.



Fig-1 Deformed shape of work pieces

Wall bracket is fixed in the wall to support the guide mechanism. The guide mechanism comprises of a pulley for lifting the weight. The set of three guide wires are provided to guide the falling weight. The weights are fixed on a disc carrying a one end thread inside the disc and other end locked to the rope by means of an eye end.

**Working:** The die holder is placed on the wooden block. The surface of the die should be free from dust, dirt and other foreign materials to provide good forming. The die is placed in the holder. The specimen is located in the shallow counter sunk bore between the die face and the cylinder. The cylinder cavity is filled with water up to desired height by removing the necessary water tap screws. Cylinder flange along with the guide bush is located by means of bolts. The plunger is inserted into the cylinder with the 'O' rings in position, up to the water level. Predetermined weight is raised to the required height manually by means of the rope over the pulley. Then it is released suddenly.



Fig-2 Experimental Setup & Multi cavity Die

The deformation of the work piece takes place by the shock waves in the water generated by the impact of the freely falling weights. The swing bolt is unfastened. The wire fixed to the cylinder flange lifts the cylinder along with the plunger, guide and guide bush. The formed specimen is removed from the shallow counter sunk bore and replaced by the new specimen. The cyclic process can be repeated by varying loads, energy input and water column heights.

The design matrix of experimentation and output are given in the Table-2

Trial No.	Design Matrix				Response (DEFORMATION) cm		
	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	Y <sub>1</sub>	Y <sub>2</sub>	Y
1	-1	-1	-1	-1	4.78	4.70	4.78
2	1	-1	-1	-1	2.95	2.89	2.95
3	-1	1	-1	-1	4.95	4.89	4.95
4	1	1	-1	-1	4.05	4.03	4.05
5	-1	-1	1	-1	5.53	5.44	5.53
6	1	-1	1	-1	4.58	4.45	4.58
7	-1	1	1	-1	5.97	5.89	5.97
8	1	1	1	-1	4.90	4.81	4.90
9	-1	-1	-1	1	5.62	5.41	5.62
10	1	-1	-1	1	3.71	3.65	3.71
11	-1	1	-1	1	5.25	5.02	5.25
12	1	1	-1	1	5.12	5.01	5.12
13	-1	-1	1	1	6.15	5.86	6.15
14	1	-1	1	1	4.52	4.4	4.52
15	-1	1	1	1	9.96	9.63	9.96
16	1	1	1	1	5.47	5.29	5.47

Table-2 Design Matrix



In the above table -1, +1 corresponding to low and high level. The values of regression coefficients are given below.  
 $b_0 = 5.1525$ ,  $b_1 = -0.7881$ ,  $b_2 = 0.4875$ ,  $b_3 = 0.6506$ ,  
 $b_4 = 0.4769$ ,  $b_{12} = -0.0169$ ,  $b_{13} = -0.2125$ ,  $b_{14} = -0.1950$ ,  
 $b_{23} = 0.1994$ ,  $b_{24} = 0.2269$ ,  $b_{34} = 0.13$ ,  $b_{123} = -0.355$ ,  
 $b_{124} = -0.1213$ ,  $b_{134} = -0.2944$ ,  $b_{234} = 0.2638$ ,  $b_{1234} = -0.2244$

The adequacy of the model was then tested by Fisher test. As per this technique, F-ratio of the model developed does not exceed the standard tabulated value of f-ratio for a 95% confidence level. Hence the model was adequate. The significance of the coefficients was checked by using student's t-test and only the significant coefficients were used to develop final mathematical model. The final models in coded form is

$$Y = 75.1525X_0 + (-0.7881)X_1 + (0.4875)X_2 + (0.6506)X_3 + (0.4769)X_4 + (0.2125)X_1X_3 + (-0.1950)X_1X_4 + (0.1994)X_2X_3 + (0.2269)X_2X_4 + (-1.15)X_3X_4 + (0.13)X_1X_2X_3 + (-0.1213)X_1X_2X_4 + (-0.2944)X_1X_3X_4 + (0.2638)X_2X_3X_4 + (-0.2244)X_1X_2X_3X_4$$

#### V. ANALYSIS OF VARIANCE

Analysis of variance is done to find out the percentage contribution of each factor and relative significance of each factor for deformation.

Table-3 ANOVA

Factor	% Contribution
X1 (Thickness of specimen)	30.2296
X2 (Height of the hammer)	11.5662
X3 (Number of blows)	20.6017
X4 (Weight)	11.0675
X1X2	0.0139
X1X3	2.1977
X1X4	1.8506
X2X3	1.9346
X2X4	2.505
X3X4	0.8225
X1X2X3	6.1334
X1X2X4	0.7155
X1X3X4	4.2174
X2X3X4	3.3855
X1X2X3X4	2.4501

#### VI. CONCLUSION

From Table-3, it is clear that the parameter 'X<sub>1</sub>', i.e., the thickness of the specimen has major influence and contribution on deformation. X<sub>2</sub> (Height of the hammer) and X<sub>4</sub> (Weight) do not contribute much to the deformation. The third parameter (X<sub>3</sub>) being the number of blows given by the hammer on to the specimen has considerable influence on it.

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