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STUDY OF FINGERO-IMBIBITION IN DOUBLE PHASE FLOW THROUGH POROUS MEDIA WITH MAGNETIC FLUID USING NUMERIC TECHNIQUE

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Abstract—**This Paper deals with the phenomena of Fingero-Imbibition in double phase flow through porous homogeneous, cracked and slightly dipping porous media involving magnetic fluid. This phenomenon arises on account of simultaneous occurrence of two important phenomena viz. imbibition and fingering. In the whole study it is assumed that injection of preferentially wetting, less viscous fluid (viz. injection fluid) into porous medium saturated with resident fluid. (viz. native fluid) is initiated under imbibition and in consequence, the resident fluid is pushed by drive in secondary recovery process. This conjoint phenomenon is known as Fingero-Imbibition.**

Keywords— Fingero-Imbibition, Porous Medium, SOR

I. INTRODUCTION

When a porous medium filled with some resident fluid is brought into contact with another fluid which preferentially wets the medium, there is a spontaneous flow of the wetting fluid into the medium and a counter flow of the resident fluid from the medium. Such a phenomena arising due to the difference in wetting abilities is called counter-current imbibition. Similarly, when a fluid contained in a porous medium is displaced by another fluid of lesser viscosity, instead of regular displacement of whole front, protuberances (fingers) may occur which shoot through the porous medium at relatively great speeds. This phenomenon is called fingering or instabilities. The phenomena of fingering and imbibition occurring simultaneously in displacement process are known as Fingero-Imbibition [1]. This phenomena gained much current importance due to their frequent occurrence in the problem of petroleum technology. Many authors have discussed them from different point of view.

In this paper, the underlying assumptions are that the two fluids are immiscible and the injected fluid is less viscous as well as preferentially wetting with respect to porous materials and with capillary pressure.

The mathematical formulation of basic equations yields a no-linear partial differential equation. A numerical solution is obtained by Successive Over Relaxation Method..

II. STATEMENT OF THE PROBLEM

We consider here a finite cylindrical mass of porous medium of length L (=1) saturated with native liquid (o), is completely surrounded by an impermeable surface except for one end of the cylinder which is labeled as the imbibition face (x=0) and this end is exposed to an adjacent formation of 'injected' liquid (w) which involves a thin layer of suitable magnetic fluid. It is assumed that the later fluid is preferentially wetting and less viscous. This arrangement gives rise to a displacement process in which the injection of the fluid (w) is initiated by imbibition and the consequent displacement of native liquid (o) produces protuberances (fingers). This arrangement describes a one – dimensional phenomenon of Fingero-Imbibition.

III. FORMULATION OF THE PROBLEM

Assuming that the flow of two immiscible phases is governed by Darcy's law, we may write the seepage velocity of injected and native fluid as,

$$V_w = -\frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x} \qquad (A)$$

$$V_o = -\frac{k_o}{\mu_o} k \frac{\partial P_o}{\partial x} \qquad (B)$$

If we take the injected liquid containing a thin layer of magnetic fluid where magnetization M is assumed to the directly proportional to the magnetic field intensity H (i.e. M= λH) and the microscopic behavior of fingers is governed by statistical treatment. Then additional pressure exerted due to pressure of a layer of magnetic fluid in the displacing liquid (w) represented by $\left[\mu_0\lambda + \frac{16\lambda\mu_0\lambda^2 r^3}{g(l+2)^3}\right]\frac{\partial H}{\partial x}$.

Therefore from (A) and (B)

$$V_w = -\left(\frac{k_w}{\mu_w}\right) k \left[\frac{\partial P_w}{\partial x} + \gamma H \frac{\partial H}{\partial x}\right] \quad (1)$$

$$V_o = -\left(\frac{k_o}{\mu_o}\right) k \left[\frac{\partial P_o}{\partial x}\right] \quad (2)$$

Where $\gamma = \mu_0\lambda + \frac{16\lambda\mu_0\lambda^2 r^3}{g(l+2)^3}$

k = Permeability of the homogeneous medium

k_w= Relative permeability of injected fluid is assumed to be functions of S_w

k_o = Relative permeabilitie of native fluid, is assumed to be functions of S_o.

P_w = Pressures of injected fluid

P_o= Pressures of native fluid

μ_o , μ_w= Constant viscosities

S_w= Saturation of of injected fluid

S_o= Saturation of native fluid

g =Acceleration due to gravity.

Neglecting the variation in phase densities, the equation of continuity for injected fluid can be written as:

$$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \quad (3)$$

$$\varphi \frac{\partial S_o}{\partial t} + \frac{\partial V_o}{\partial x} = 0 \quad (4)$$

Where φ is the porosity of the medium.

From the definition of phase saturation, it is obvious that

$$S_w + S_o = 1 \quad (5)$$

The analytic condition (Scheidegger, 1960) [2] governing imbibition phenemenon is

$$V_w + V_o = 0 \quad (6)$$

The capillary pressure (P_c), defined as the pressure discontinuity of the flowing phases across their common interface, may be written as :

$$P_c = P_o - P_w \quad (7)$$

Substituting (1) and (2) in (6), we have

$$\left(\frac{k_w}{\mu_w}\right) k \left[\frac{\partial P_w}{\partial x} + \gamma H \frac{\partial H}{\partial x}\right] + \left(\frac{k_o}{\mu_o}\right) k \left[\frac{\partial P_o}{\partial x}\right] = 0 \quad (8)$$

Using (7), equation (8) reduces to the form,

$$\left(\frac{k_w}{\mu_w}\right) k \left[\frac{\partial P_w}{\partial x} + \gamma H \frac{\partial H}{\partial x}\right] + \left(\frac{k_o}{\mu_o}\right) k \left[\frac{\partial P_c}{\partial x} + \frac{\partial P_w}{\partial x}\right] = 0 \quad (9)$$

$$\frac{\partial P_w}{\partial x} = -\left[\frac{\left(\frac{k_o}{\mu_o}\right)\frac{\partial P_c}{\partial x} + \left(\frac{k_w}{\mu_w}\right)\gamma H \frac{\partial H}{\partial x}}{\left(\frac{k_o}{\mu_o} + \frac{k_w}{\mu_w}\right)}\right] \quad (10)$$

From (1) and (10), we get,

$$V_w = \left(\frac{k_w k_o}{\mu_o k_w + \mu_w k_o}\right) k \left[\frac{\partial P_c}{\partial x} - \gamma H \frac{\partial H}{\partial x}\right] \quad (11)$$

From (3) and (11), we get,

$$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left[\left(\frac{k_w k_o}{\mu_o k_w + \mu_w k_o}\right) k \left[\frac{\partial P_c}{\partial x} - \gamma H \frac{\partial H}{\partial x}\right]\right] = 0 \quad (12)$$

This is the desired non-linear partial differential equation describing the fingero-imibibition phenomenon for the flow of two immiscible phases through porous media. Since the present investigation involves injected fluid and a viscous native fluid, therefore according to Scheidegger (1960) approximation, we may write eq. (12) in the form

$$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left[\left(\frac{k_o}{\mu_o}\right) k \left[\frac{\partial P_c}{\partial x} - \gamma H \frac{\partial H}{\partial x}\right]\right] = 0 \quad (13)$$

As $\frac{k_w k_o}{\mu_o k_w + \mu_w k_o} \approx \frac{k_o}{\mu_o}$

At this state, for definiteness of the mathematical analysis, we assume that standard relationship due to Scheidegger and Johnson [3], Muskat [4], between phase saturation and relative permeability as

$$k_w = S_w, \; k_o = S_o = 1 - S_w, \; P_c = -\beta S_w \quad (14)$$

Where β is the capillary pressure co-efficient.

From (13) and (14), we get

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$$\varphi\frac{\partial S_w}{\partial t}+\frac{k}{\mu_o}\frac{\partial}{\partial x}\left[(1-S_w)\left[-\beta\frac{\partial S_w}{\partial x}-\gamma H\frac{\partial H}{\partial x}\right]\right]=0 \qquad (15)$$

Considering the magnetic fluid H in the x-direction only, we may write (3), $H=\frac{\Lambda}{x^n}$, where Λ is a constant parameter and n is an integer.Using the value of H for $n=-1$ in eq.(15), we get,

$$\varphi\frac{\partial S_w}{\partial t}+\frac{k}{\mu_o}\frac{\partial}{\partial x}\left[(1-S_w)\left[-\beta\frac{\partial S_w}{\partial x}-\gamma\Lambda^2 x\right]\right]=0$$

or

$$\varphi\frac{\partial S_w}{\partial t}-\frac{k\beta}{\mu_o}\frac{\partial}{\partial x}\left[(1-S_w)\frac{\partial S_w}{\partial x}\right]-\frac{k\gamma\Lambda^2}{\mu_o}\frac{\partial}{\partial x}[(1-S_w)x]=0 \qquad (16)$$

A set of suitable initial and boundary conditions associated to problem (16) are

$$s_w(x,0)=s_c,\ \text{for } x>0 \qquad (17)$$

$$s_w(0,t)=s_0;\ s^*_w(L,t)=s_1 \text{ for } t\geq 0 \qquad (18)$$

Equation (18) is reduced to dimensional form by setting

$$X=\frac{x}{L},\quad T=\frac{k\beta}{\mu_o L^2\varphi}t,\ s^*_w(X,t)=1-s_w(x,t)$$

And then eq. (18) takes the form

$$\frac{\partial S_w}{\partial T}=\frac{\partial}{\partial X}\left[S_w\frac{\partial S_w}{\partial X}\right]-C_0\frac{\partial}{\partial X}[S_w X] \qquad (19)$$

Where $C_0=\frac{\gamma\Lambda^2L^2}{\mu_o}$

$$\frac{\partial S_w}{\partial T}=\frac{\partial}{\partial X}\left[S_w\frac{\partial S_w}{\partial X}\right]-C_0X\frac{\partial S_w}{\partial X}-C_0S_w \qquad (20)$$

With boundary conditions

$$s_w(X,0)=1-s_c,\ \text{for } 0<X\leq L \qquad (21)$$

$$s_w(0,T)=1-s_0;\ s_w(L,t)=1-s_1 \text{ for } t\geq 0 \qquad (22)$$

In eq. (20), the asterisk is dropped for simplicity. Eq. (20) is desired non-linear partial differential equation for the flow of two immiscible liquids in homogeneous medium with effect of magnetic fluid.

IV. MATHEMATICAL SOLUTION

Using finite differences for (20),

$$S_{w_{m,n+1}}$$

$$=(1-kc_0)S_{w_{m,n}}+\frac{r}{2}\left[S^2{}_{w_{m+1,n}}-2S^2{}_{w_{m,n}}+S^2{}_{w_{m-1,n}}\right]-\frac{c_0rhx_m}{2}\left(S_{w_{m+1,n}}-S_{w_{m-1,n}}\right)$$

Using Gauss-Seidal Method,

$$S_{w_{m,n+1}}$$

$$=(1-kc_0)S_{w_{m,n}}+\frac{r}{2}\left[S^2{}_{w_{m+1,n}}-2S^2{}_{w_{m,n}}+S^2{}_{w_{m-1,n+1}}\right]-\frac{c_0rhx_m}{2}\left(S_{w_{m+1,n}}+S_{w_{m-1,n+1}}\right) \qquad (23)$$

Using Successive –Over Relaxation Method,

Let

$$C_m=(1-kc_0)S_{w_{m,n}}+\frac{r}{2}\left[S^2{}_{w_{m+1,n}}-2S^2{}_{w_{m,n}}\right]-\frac{c_0rhx_m}{2}S_{w_{m+1,n}}$$

From (23) we have,

$$S_{w_{m,n+1}}=C_m+\frac{r}{2}S^2{}_{w_{m-1,n+1}}+\frac{c_0rhx_m}{2}S_{w_{m-1,n+1}}$$

$$S_{w_{m,n+1}}$$

$$=S_{w_{m,n}}+\omega\left[C_m+\frac{r}{2}S^2{}_{w_{m-1,n+1}}+\frac{c_0rhx_m}{2}S_{w_{m-1,n+1}}-S_{w_{m,n}}\right] \qquad (24)$$

Let k = 0.01. h = 0.1, $c_0=0.85$, $\omega=1.7$

From (24) we have

$$S_{w_{m,n+1}}$$

$$=-0.7S_{w_{m,n}}+1.7\left[C_m+0.5S^2{}_{w_{m-1,n+1}}+0.0425x_mS_{w_{m-1,n+1}}\right]$$

Numerical values are shown by the following table.

	T =0.01	T=0.02	T =0.03	T=0.04
x	S_w			
0.	0.5	0.5	0.5	0.5
0.	0.2161125	0.3509535	0.3655792	0.3885172
0.	0.0428217	0.1488194	0.2280581	0.27365482
0.	0.0024868	0.0244896	0.0722417	0.13179382
0.	0.0000771	0.0012927	0.0077935	0.02613975
0.	0.0000027	0.0000507	0.0003831	0.00190174
0.	0.0000001	0.0000023	0.0000190	0.00010421
0.	0.0000000	0.0000001	0.0000010	0.0.0000077
0.	0.0000000	0.0000000	0.0000000	0.00000053
0.	0.0	0.0	0.0000001	0.0000004

1.	0.0	0.0	0.0	0.0

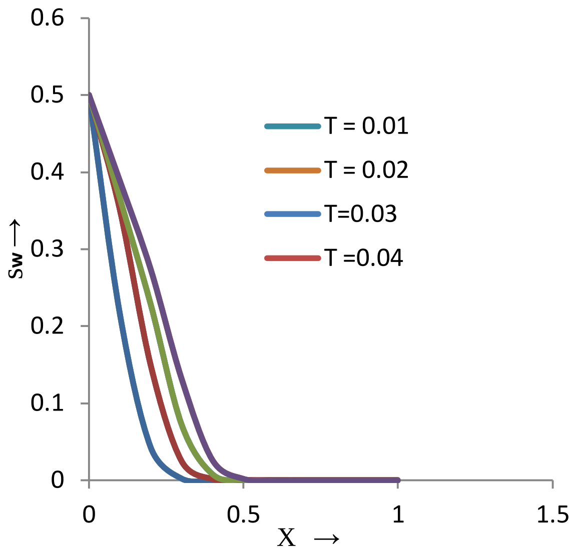

Figure –a: Length (X) → Saturation(S_W)

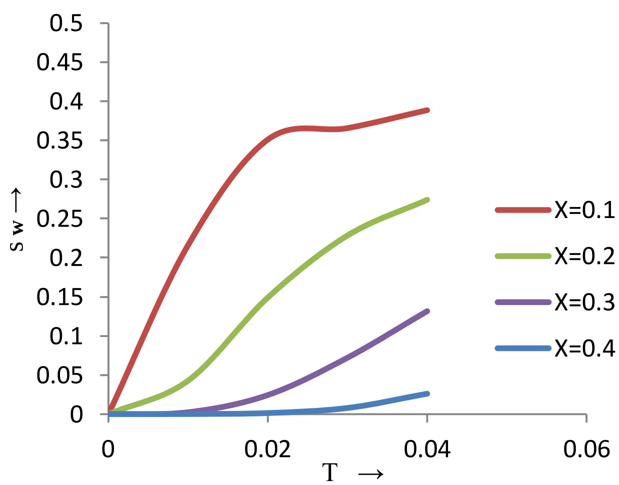

Figure – b :Time (T) → Saturation(S_W)

V. INTERPRETATION

From figure-a, we can say that as X increases the saturation (S_w) decreases. Keeping T constant as X increases saturation decreases parobolically. From figure-b, it is clear that as time (T) increases saturation (S_w) increases. Keeping X constant, for different values of (T) the saturation (S_w) increases parabolically as time (T) increases. From both the graphs, it is clear that the curves are of parabolic type. Again it is clear that the governing equation is parabolic.

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